Department of Economic Policy
Working Paper Series

WP No. 20

Pools or Schools or Both?
Diversification and Specialisation in Public Services

Authors: Dujava, D. – Černěnko, T. – Rafaj, O.

Date: September 30, 2019

Published by:

University of Economics in Bratislava, Department of Economic Policy, Dolnozemská cesta 1, 852 35 Bratislava

Periodicity: irregular

Peer-reviewed

ISSN 1339 - 0430

Pools or Schools or Both? Diversification and Specialisation in Public Services*

Daniel Dujava[†a], Tomáš Černěnko[‡a], and Oliver Rafaj[§a]

[a]Faculty of National Economy, University of Economics in Bratislava

September 30, 2019

Abstract

We develop a model of two municipalities providing two public services. We show that in case of intermediate level of transferability of public services between two municipalities, Pareto-suboptimal Nash equilibrium exists. In this case, cooperation between municipalities or merging of two municipalities into one unit would improve welfare.

Keywords: diversification, game theory, public services, specialisation

JEL Codes: C72, H11

*We would like to thank Richard Kališ for his helpful comments and insights.

[†]e-mail:daniel.dujava@euba.sk
[‡]e-mail:tomas.cernenko@euba.sk
[§]e-mail:oliver.rafaj@euba.sk

1 Introduction

Two of the most important characteristics in production of many public services (and goods) are their nonrivalrous nature and the fact that producer of public services attempts to maximize *public* welfare, not *private* profit. If public services are provided on municipal level, a town can maximize utility of its inhabitants only if it takes into consideration set of public services provided by neighbouring municipalities. Consider the following example: Why should town A invest in public swimming pool if a pool is already available in neighbouring town B? Wouldn't it be more efficient to invest in a local high school? Children commuting from B to A would benefit as well.

However, even though 'looking over the border' is necessary for welfare maximisation, it is not sufficient. Isn't it possible that lack of cooperation between municipalities will lead to sub-optimal decisions despite the best efforts of each decision making unit to improve welfare of its inhabitants, especially if number of municipalities is high[1]? In this article we conceptualize this question and provide simple analytical approach enabling to think about problem of specialisation and diversification in public services provision on local level.

We show that if costs of transferring public services between two municipalities are at intermediate level (their are non-negligible, but they are not prohibitive), it is possible that municipalities will end up in Pareto-suboptimal equilibrium. In this case, cooperation or merging of municipalities will lead to superior allocation of resources. Note that merging of municipalities is exactly what has been recommended to Slovakia and other countries in the region from various sources, for example in Swianiewicz (2010); Nemec and de Vries (2015) or Cernenko et al. (2017).

To the best of our knowledge, problem of specialisation and diversification in production of public services was not addressed in the literature using game-theoretic approach. In this way, we contribute to the literature.

2 The Model

2.1 Technology and preferences

Assume municipalities A and B producing public services X and Y. Municipality i chooses non-negative *quality* of service X_i and Y_i (i.e. respecting $X_i \in [0,1]$ and $Y_i \in [0,1]$) under the fixed budget constraint

[1] The administrative structure in Slovakia is one of the most fragmented in Europe, the number of mayors per 100,000 inhabitants ranks 3rd among European countries right behind Czech Republic and France, see Cernenko et al. (2017)

$$X_i + Y_i = 1, \tag{1}$$

where $i \in \{A, B\}^2$. Choosing $X_i = 0$ (or $Y_i = 0$) is equivalent to not producing X (or Y) at all.

Consumers choose whether to use service provided in their home municipality or whether to commute to the neighbouring town. Assume that consumers in municipality i are indifferent between consuming services provided in home town i and commuting to neighbour town i' if $S_i = aS_{i'}$, $S \in \{X, Y\}$. Parameter $0 < a < 1$ describes degree of transferability of public services between two towns, $1 - a$ can be interpreted as transport costs. If $S_i > aS_{i'}$ citizens consume S in their home town i, if $S_i < aS_{i'}$ citizens commute into i'.

Both services X and Y are non-rival, i.e. quality (and quantity) of services available to citizens in municipailty i do not change if citizens from neighbour town i' choose to commute and consume services in i.

Assuming separable logarithmic utility function, utility of citizens in municipality i is given by:

$$U_i = \log\left[\max\left(X_i, aX_{i'}\right)\right] + \log\left[\max\left(Y_i, aY_{i'}\right)\right], \tag{2}$$

Municipality i maximizes (2) with respect to (1) taking actions of other town i' as given.

Using (1), utility of citizens in municipality i (2) can be rewritten as a function of quality of service X provided in towns i and i':

$$U_i(X_i, X_{i'}) = \begin{cases} \log aX_{i'} + \log\left(1 - X_i\right), & X_i \in [0, aX_{i'}] \\ \log X_i + \log\left(1 - X_i\right), & X_i \in [aX_{i'}, 1 - a + aX_{i'}] \\ \log X_i + \log\left[a\left(1 - X_{i'}\right)\right], & X_i \in [1 - a + aX_{i'}, 1] \end{cases} \tag{3}$$

Figure 1 plots utility U_i against X_i for different values of a assuming $X_{i'} = 0.4$.

Observe that the utility function is decreasing in X_i on the left interval $X_i \in [0, aX_{i'}]$ and increasing in X_i on the right interval $X_i \in [1 - a + aX_{i'}, 1]$. On the middle interval $X_i \in [aX_{i'}, 1 - a + aX_{i'}]$ the utility function is increasing if $X_i < 0.5$ and decreasing if $X_i > 0.5$. In other words, assuming that 0.5 falls into the interval $[aX_{i'}, 1 - a + aX_{i'}]$, on this interval utility function has an inverse U-shape given by $\log X_i + \log\left(1 - X_i\right)$, as depicted in Figure 1.

[2] More generally, budget constraint should be expressed as $X_i + Y_i \leq 1$. However, it is easy to see that municipality can always increase utility of citizens by increasing quality of services X or Y. Budget constraint is therefore always binding.

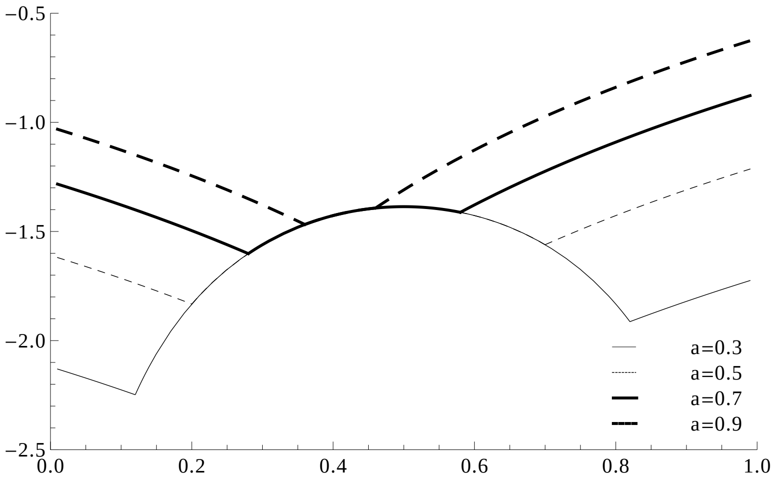

Figure 1: Utility function $U_i(X_i, 0.4)$ for different a's

2.2 Unilateral utility maximizing and policy mapping

The most important feature of the utility function is that **there are either two or three local maxima** depending on values of a and $X_{i'}$. Those are at (i) $X_i = 0$, (ii) $X_i = 1$ and if $aX_{i'} < 0.5 < 1 - a - aX_{i'}$ there is the third local maximum at (iii) $X_i = 0.5$.

Global maximum is obtained by **comparing value of utility function at local maxima** and picking between $X_i = 0$, $X_i = 0.5$ and $X_i = 1$. It is possible that there are several choices of X_i which maximize the utility function. Observe that $U_i(0, X_{i'}) = \log aX_{i'} + \log(1 - 0) = \log aX_{i'}$, $U_i(1, X_{i'}) = \log 1 + \log[a(1 - X_{i'})] = \log[a(1 - X_{i'})]$ and if there is the third maximum then $U_i(0.5, X_{i'}) = \log 0.5 + \log 0.5 = \log 0.25$.

It is useful to distinguish three cases:

Case 1 (low transferability): $a < 0.25$

In this cases, all three maxima need to be compared. Since $X_{i'} \in [0, 1]$, $U_i(0.5, X_{i'}) = \log 0.25 > U_i(0, X_{i'}) = \log aX_{i'}$ as well as $U_i(0.5, X_{i'}) = \log 0.25 > U_i(1, X_{i'}) = \log[a(1 - X_{i'})]$. Therefore, maximum utility is always obtained for $X_i^* = 0.5$. In case of low transferability of public services, municipality i always diversifies. Table 1 gives numerical example.

Case 2 (intermediate transferability): $0.25 \leq a \leq 0.5$

In this case, also all three maxima need to be compared and it is easy to see that (i) if $X_{i'} \leq 1 - 0.25/a$, $X_i^* = 1$ is optimal solution, (ii) if $1 - 0.25/a \leq X_{i'} \leq 0.25/a$, $X_i^* = 0.5$ is utility maximizing and if (iii) $0.25/a \leq X_{i'}$, $X_i^* = 0$ is optimal (for a numerical example, see Table 1). Loosely speaking, town i specializes if town i' specializes and it diversifies if neighbouring town diversifies.

Note that it is possible that multiple solutions are utility-maximizing. This is the case (i) if $X_{i'} = 1 - 0.25/a$ (both $X_i^* = 1$ and $X_i^* = 0.5$ are optimal), (ii) if $X_{i'} = 0.25/a$ (both $X_i^* = 0.5$ and $X_i^* = 1$ are optimal) and (iii) if $a = 0.5$ and $X_{i'} = 0.25/a = 1 - 0.25/a = 0.5$ (all three $X_i^* = 0$, $X_i^* = 0.5$ and $X_i^* = 1$ are optimal).

Case 3 (high transferability): $0.5 < a < 1$

In this case either $U_i(0, X_{i'}) = \log aX_{i'} > \log 0.25 = U_i(0.5, X_{i'})$ or $U_i(1, X_{i'}) = \log [a\,(1 - X_{i'})] > \log 0.25 = U_i(0.5, X_{i'})$. Therefore, only two possible solutions need two be taken into consideration. Municipality i fully focuses either on X or on Y. In particular, if $X_i \leq 0.5$, U_i is maximized for $X_i^* = 1$, if $0.5 \leq X_i$, U_i is maximized for $X_i^* = 0$ (numerical example can be found in Table 1). In other words, if transferability of public services is high, municipality i chooses to focus solely on one public service - the one not provided (or provided in low quality) by neighbouring town.

Existence of multiple solutions is also possible. If $X_{i'} = 0.5$, both $X_i^* = 0$ and $X_i^* = 1$ are optimal.

Table 1: Finding utility maximizing X_i: Numerical example

a	$X_{i'}$	Spec. on Y: $X_i = 0$	Div.: $X_i = 0.5$	Spec. on X: $X_i = 1$
$a = 0.2$ (case 1)	$X_i' = 0.1$	$U_i = \log aX_{i'} = \log 0.02$	$U_i = \mathbf{\log 0.25}$	$U_i = \log [a\,(1 - X_{i'})] = \log 0.18$
	$X_i' = 0.5$	$U_i = \log aX_{i'} = \log 0.10$	$U_i = \mathbf{\log 0.25}$	$U_i = \log [a\,(1 - X_{i'})] = \log 0.10$
	$X_i' = 0.9$	$U_i = \log aX_{i'} = \log 0.18$	$U_i = \mathbf{\log 0.25}$	$U_i = \log [a\,(1 - X_{i'})] = \log 0.02$
$a = 0.4$ (case 2)	$X_i' = 0.1$	$U_i = \log aX_{i'} = \log 0.04$	$U_i = \log 0.25$	$U_i = \log [a\,(1 - X_{i'})] = \mathbf{\log 0.36}$
	$X_i' = 0.5$	$U_i = \log aX_{i'} = \log 0.02$	$U_i = \mathbf{\log 0.25}$	$U_i = \log [a\,(1 - X_{i'})] = \log 0.02$
	$X_i' = 0.9$	$U_i = \log aX_{i'} = \mathbf{\log 0.36}$	$U_i = \log 0.25$	$U_i = \log [a\,(1 - X_{i'})] = \log 0.02$
$a = 0.6$ (case 3)	$X_i' = 0.1$	$U_i = \log aX_{i'} = \log 0.06$	$U_i = \log 0.27$	$U_i = \log [a\,(1 - X_{i'})] = \mathbf{\log 0.54}$
	$X_i' = 0.5$	$U_i = \log aX_{i'} = \mathbf{\log 0.30}$	$U_i = \log 0.25$	$U_i = \log [a\,(1 - X_{i'})] = \mathbf{\log 0.30}$
	$X_i' = 0.9$	$U_i = \log aX_{i'} = \mathbf{\log 0.54}$	$U_i = \log 0.27$	$U_i = \log [a\,(1 - X_{i'})] = \log 0.06$

Note: Utility-maximizing choice in bold.

It follows that policy mapping $X_i^* = h(X_{i'})$ giving set of utility-maximizing choices of X_i^* based on $X_{i'}$ is given by:

$$X_i^* = h(X_{i'}) = \begin{cases} 1, & (X_{i'} \leq 1 - 0.25/a \wedge a < 0.5) \vee (X_{i'} \leq 0.5 \wedge 0.5 \leq a) \\ 0.5, & 1 - 0.25/a \leq X_{i'} \leq 0.25/a \\ 0, & (0.25/a \leq X_{i'} \wedge a < 0.5) \vee (0.5 \leq X_{i'} \wedge 0.5 \leq a) \end{cases} \quad (4)$$

Policy mapping is depicted in Figures 2-7, $X_{i'}$ on horizontal axis, X_i^* on vertical axis. Note that in several cases there are multiple X_i^* maximizing the utility in municipality i.

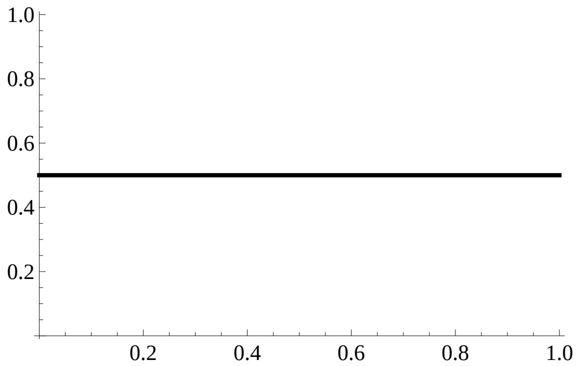

Figure 2: P. mapping $X_i^* = h(X_{i'})$, $a < 0.25$

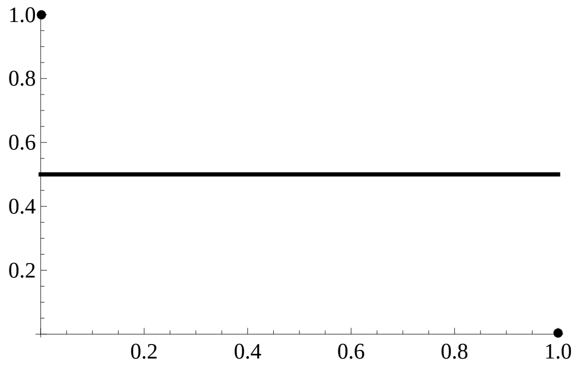

Figure 3: P. mapping $X_i^* = h(X_{i'})$, $a = 0.25$

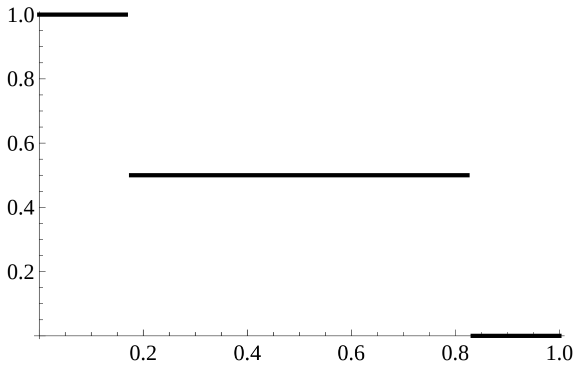

Figure 4: P. mapping $X_i^* = h(X_{i'})$, $= 0.3$

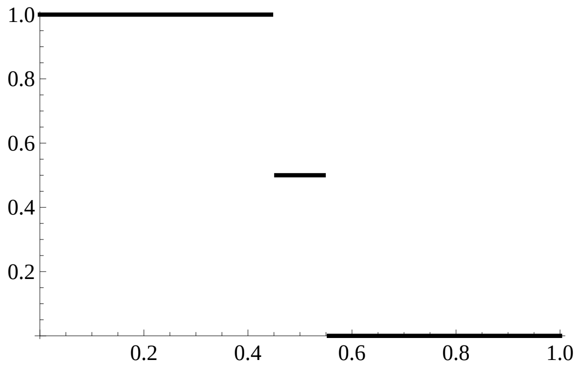

Figure 5: P. mapping $X_i^* = h(X_{i'})$, $a = 0.45$

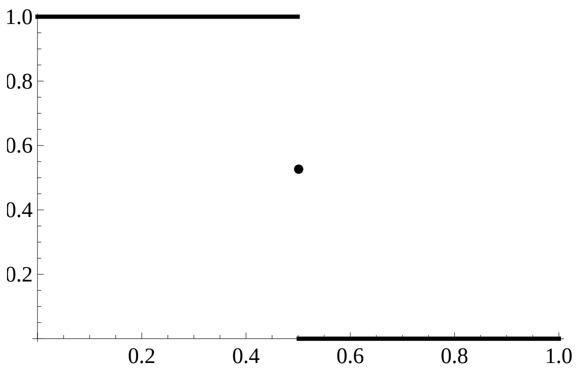

Figure 6: P. mapping $X_i^* = h(X_{i'})$, $a = 0.5$

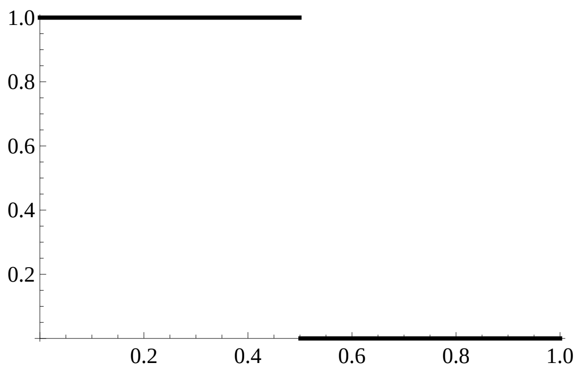

Figure 7: P. mapping $X_i^* = h(X_{i'})$, $a > 0.5$

2.3 Nash equilibria and Pareto optimality

Nash equilibrium $\{X_A^*, X_B^*\}$ requires $X_A^* = h(X_B^*)$ and $X_B^* = h(X_A^*)$. Once again, distinguish between three cases:

Case 1 (low transferability): $a < 0.25$

Since in this case only $X_i^* = 0.5$ maximizes U_i irrespective of $X_{i'}$, there is only one Nash equilibrium:.

1. Both municipalities divesify, i.e. $\{X_A^*, Y_A^*, X_B^*, Y_B^*\} = \{0.5, 0.5, 0.5, 0.5\}$.

It is easy to show that this solution is also a single Pareto-optimal outcome. To see this, maximize first U_A with respect to *both* X_A and X_B.

To find the maximum $\{X_A^{max}, X_B^{max}\}$, proceed in two steps: First, use the fact that for a given $X_{i'}$, $U(X_i^{max}, X_{i'}) = \max\{\log aX_{i'}, \log 0.25, \log[a(1 - X_{i'})]\}$. To maximize $U(X_i^{max}, X_{i'})$ with respect to $X_{i'}$ it is sufficient to find $X_{i'} \in [0,1]$ which maximizes expression $\max[aX_{i'}, 0.25, (1 - X_{i'})]$. Second, X_i^{max} is given by policy mapping (4).

In this case, maximizing U_A yields maximum at $X_A^{max} = 0.5$ irrespective of X_B and $U_A(0.5, X_B) = \log 0.25$. Analogically, U_B is maximized at $X_B^{max} = 0.5$ irrespective of X_A and $U_B(0.5, X_A) = \log 0.25$. Allocation $\{X_A^*, Y_A^*, X_B^*, Y_B^*\} = \{0.5, 0.5, 0.5, 0.5\}$ is therefore Pareto-superior to all other allocations.

Summing up, if it is costly to commute to neighbouring municipality, both towns will diversify and will produce both public services of average quality what is an optimal outcome.

Case 2 (intermediate transferability): $0.25 \leq a \leq 0.5$

In this case there are three possible Nash equilibria:

1. Municipality A specializes on X, municipality B specializes on Y, i.e. $\{X_A^*, Y_A^*, X_B^*, Y_B^*\} = \{1, 0, 0, 1\}$.
2. Municipality A specializes on Y, municipality B specializes on X, i.e. $\{X_A^*, Y_A^*, X_B^*, Y_B^*\} = \{0, 1, 1, 0\}$.
3. Both municipalities diversify, i.e. $\{X_A^*, Y_A^*, X_B^*, Y_B^*\} = \{0.5, 0.5, 0.5, 0.5\}$.

Pareto-optimality can be again investigated by maximizing U_A with respect to both X_A and X_B.

First, assume $a = 0.25$. Three maxima are obtained at $\{X_A^{max}, X_B^{max}\} = \{1, 0\}$, $\{X_A^{max}, X_B^{max}\} = \{0.5, 0.5\}$ and $\{X_A^{max}, X_B^{max}\} = \{0, 1\}$. In all allocations, $U_A = \log 0.25$. Maximizing utility of citizens in town B with respect to both X_A and X_B yields same allocations and in each $U_B = \log 0.25$. It follows that three Nash equilibria are Pareto-superior to all other allocations and are therefore Pareto-optimal.

Second, assume $0.25 < a \leq 0.5$ and follow the same steps maximizing U_A with respect to both X_A and X_B. *Only two maxima* are obtained at $\{X_A^{max}, X_B^{max}\} = \{1, 0\}$ and $\{X_A^{max}, X_B^{max}\} = \{0, 1\}$, in both allocations $U_A = \log a$. Utility U_B is maximized at same allocations and $U_B = \log a$. It follows that if $0.25 < a \leq 0.5$ specialization is Pareto-superior to diversification even though both are equilibrium outcomes.

In case of intermediate transferability, it is possible that municipalities find themselves in sub-optimal equilibrium in which they both produce both public services of average quality even though diversification would provide higher utility for citizens. However, neither municipality has incentive to change the structure of provided services first. Joint action is necessary to break the deadlock.

Case 3 (high transferability): $0.5 < a < 1$

In case of high transferability, diversification is never utility-maximizing choice. Therefore, there are two possible Nash equilibria:

1. Municipality A specializes on X, municipality B specializes on Y, i.e. $\{X_A^*, Y_A^*, X_B^*, Y_B^*\} = \{1, 0, 0, 1\}$.
2. Municipality A specializes on Y, municipality B specializes on X, i.e. $\{X_A^*, Y_A^*, X_B^*, Y_B^*\} = \{0, 1, 1, 0\}$.

To see that both Nash equilibria are Pareto-optimal, maximize once again U_A with respect to both X_A and X_B. Two maxima are obtained at $\{X_A^{max}, X_B^{max}\} = \{1, 0\}$ and $\{X_A^{max}, X_B^{max}\} = \{0, 1\}$. In both allocations, $U_A = \log a$. Utility U_B is maximized at same allocations and in both $U_B = \log a$. Therefore, Nash equilibria in which municipalities diversify are Pareto-superior to all other allocations.

If there are little costs associated with commuting to neighbouring municipality, both will specialize on production of single public service what is an optimal outcome.

3 Conclusion

We have provided a simple analytical approach to issue of specialisation and diversification in production of public services showed that in case of intermediate transferability of public services between two municipalities, suboptimal equilibrium can arise. Extension two cases of more than two municipalities as well as empirical relevancy of this scenario will be subject of further research.

References

Cernenko, T., P. Havran, and J. Kubala (2017). Skryty poklad v samosprave: alternativne moznosti sustredenia vykonu spravy v samospravach. Publikacie IFP, Institut financnej politiky a Narodohospodarska fakulta Ekonomickej univerzity v Bratislave.

Nemec, J. and M. S. de Vries (2015). Local Government Structure and Capacities in Europe. *Public Policy and Administration 14*(3), 249–267.

Swianiewicz, P. (2010). If Territorial Fragmentation is a Problem, is Amalgamation a Solution? An East European Perspective. *Local Government Studies 36*(2), 183–203.