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Economic growth and convergence during the transition to production using automation capital*

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Abstract

This paper examines the implications of automation capital in a Solow growth model with two types of labour. We study the transition from standard production to production using automation capital which substitutes low-skilled workers. We assume that despite advances in technology, AI and machine learning, certain tasks can be performed only by high-skilled labour and are not automatable. We show that under these assumptions, automation capital does not generate endogenous growth without technological progress. However, assuming presence of technological progress augmenting both effective number of workers and effective number of industrial robots, automation increases rate of long-run growth. We analyse a situation in which some countries do not use robots at all and other group of countries starts the transition to the economy where industrial robots replace low-skilled labour. We show that this has potential non-linear effects on σ-convergence and that the model is consistent with temporary divergence of incomes per capita. We derive a set of estimable equations that allows us to test the hypotheses in a Mankiw-Romer-Weil framework.

Keywords: Automation, Economic growth, Income inequality, Convergence, Robots

JEL Codes: D63, E25, O11, O41

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1 Introduction

Increased usage of industrial robots and other forms of automation capital in production motivated the construction of economic growth models with automation-driven long-run growth. There is a variety of economic models with automation capital that capture different aspects of the economic consequences. What they share in common is their emphasis on the potential of automation capital to generate constant returns to scale to producible inputs and thus generate long-run economic growth without the technological progress. Little attention is paid to a situation currently faced by the most countries in the world. That is, the transition from an economy where automation capital is not present or plays a marginal role, towards the economy that invests increasing proportion of savings to automation capital. Thus, we contribute to the current literature by providing a simple framework that allows as to study the implications of a transition to economy with automation capital for economic growth, convergence process and income inequality. We study the situation without exogenous technological progress in which automation capital does not generate perpetual growth. However, we show that assuming technological progress which augments both effective number of labour *and* effective number of industrial robots, automation capital increases the long-run growth. We derive a set of estimable equations which allow us to test hypotheses suggested by the model in a Mankiw-Romer-Weil framework. We confirm the hypothesis that the speed of convergence towards a steady state in an economy with robots is lower than the convergence towards a pseudo steady state where robots are not employed in the production. We also find evidence in favour of the hypothesis that elasticity of output with respect to savings rate is higher if robots are used in the production. Furthermore, we show that assuming that i) countries differ in fundamentals (for example in total factor productivity) and ii) there is a convergence in fundamentals (for example differences in TFP are decreasing), it is possible that coefficient of σ-convergence evolves in non-linear fashion. In particular, we show that robot-technology can be a cause of temporary divergence in output per capita what is consistent with the data.

In Figure 1 we show the development of σ-convergence, overall stock of robots and the share of countries with robots in a sample of 65 countries. The standard deviation of incomes per capita was decreasing until 1996, increasing in the period from 1996 to 2001 and decreasing again thereafter. This development coincides with an increase of the share of countries producing with automation capital from 40 % in 1993 to more than 90 % in 2014. Not only the share of countries using robots increased during this period but the overall stock of robots in the world economy rose dramatically. Total stock of robots more than tripled from 1993 to 2014.

The paper is structured as follows: First, we review the current literature on models with automation capital and show our contribution. In Section 3 we propose a Solow-type growth model with two types of labour, traditional and automation capital. We derive the model without population growth and without technological progress. Here we analyse the transitional dynamics from a situation without robots towards the production based on the introduction of robots as additional factor of production. Population growth and technological progress are introduced into the model in Section 4. We show that wages of low-skilled workers are stagnant despite exogenous technological progress and that the model is able to generate non-linear development of coefficient of σ-convergence. In Section 5 we derive a set of equations for empirical analysis, we present an empirical strategy and the first empirical results which confirm hypotheses suggested by the model.

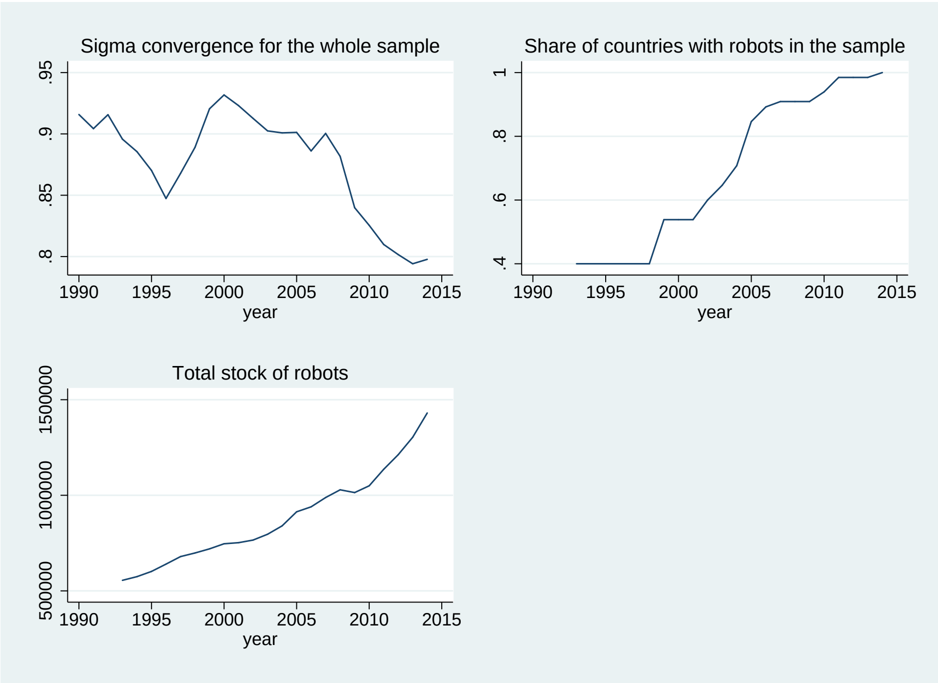

Figure 1: σ-convergence, share of countries with robots and total stock of robots

2 Literature Review

Steigum (2011) was the first to investigate the automation capital in a one-sector Cass-Koopmans-Ramsey model of optimal growth. He showed that robots can become a source of endogenous growth some time in the future. Robots as a perfect substitute for labour where first introduced to Solow model by Prettner (2017). Prettner (2017) shows that due to automation capital: i) perpetual growth is possible even in the absence of technological progress; ii) the long-run economic growth rate declines with population growth; iii) there is a unique share of savings diverted to automation that maximizes the long-run growth rate of the economy; and iv) the labour share declines with automation.

Gasteiger and Prettner (2017) analyse long-run growth effects of automation in the canonical overlapping generations framework. They show that standard neoclassical growth models (Solow, 1956; Cass, 1965; Koopmans, 1965) and Diamond´s overlapping generations model (Diamond, 1965) lead to opposite predictions with regards to the growth effects of automation.

In the first class of models, households save a part of their wage income and a part of their asset income. This implies that automation could lead to perpetual long-run growth even without (exogenous or endogenous) technological progress. On the other side, in the framework of Diamond (1965) households save exclusively out of wage income. Because automation capital competes closely with workers, an increase in the stock of automation capital does not raise worker´s productivity measured by their marginal value product. Thus, automation capital depresses the wages and therefore the prospects for future growth as well.

The first generation of growth models with automation does not differentiate among workers. But while some fraction of workers can be replaced by automation capital (they are substitutes), the other fraction of workers cooperates with and maintains the automation capital (they are complements). Lankisch et al. (2019) generalize the results of Prettner (2017). They aim to explain the simultaneous presence of increasing output per capita, declining real wages of low-skilled workers, and a rising wage premium of higher education within a model of economic growth in the age of automation. They work with nested CES production function with automation capital as a perfect substitute for low-skilled labor, assuming low-skilled labor and high-skilled labor are gross substitutes. Thus, automation capital competes directly with low-skilled labor and indirectly with high-skilled workers [1]. Accumulation of automation capital leads to diminishing wages of low-skilled workers and increases the wages of high-skilled workers. Skill premium rises as a consequence of increased stock of automation capital [2]. The implications of automation have been studied within the R&D based endogenous growth models by Acemoglu and Restrepo (2018), Hémous and Olsen (2014), Prettner and Strulik (2017).

Recent models with automation capital show its possibilities to generate growth witout technological progress. Automation capital thus complements the list of other potential sources of endogenous growth. The basic implications of most models with automation capital do not change in a presence of exogenous technological progress. Our model differs in two dimensions:

First, we explicitly assume that high-skilled workers are a necessary input and *cannot* be substituted by industrial robots. We make this assumption because our focus is on current transitional period during which automation capital is used to perform certain activities, but some activities still remain exclusive domain of humans. We argue that this framework is more suitable when studying current economic phenomena, whereas assumption of full substitutability of both low-skilled and high-skilled labour by automation capital is more reasonable when investigating more distant future. Due to the assumption that high-skilled labour is necessary input, our model does not generate endogenous growth without technological progress. Technological progress remains exogenous in our model.

[1] They endogenize the decision of rational investors of how much to invest in the two different stocks of capital. They abstract from technological progress (no change in central results). There are two steady states, for low savings a standard no growth steady state, but perpetual balance growth path is also possible.

[2] They show that it potentially diminishes wages of high-skilled workers in a situation with high substitutability between low- and high- skilled workers.

Second, we introduce technological progress which augments both effective number of workers as well as effective number of industrial robots (it seems reasonable to call it labour- *and robot*-augmenting technological progress).

To our knowledge, we are the first to explicitly study a situation in which only some fraction of economies adopts robots even though this situation is possible in some of the models proposed in the literature. We explicitly control for different efficiency of robots that are replacing the low-skilled labour. It is common in recent papers to assume that one robot replaces one low-skilled worker or that this ratio is fixed. But robots can became more efficient in terms of replacing labour, they can operate 24 hours, and over time one robot can perform increasing spectrum of operations. Therefore, we also consider exogenous shifts in robots' efficiency.

3 A model without population growth and technological progress

Assume production function:

$$Y_t = F(L_H, L_L, P_t, K_t) = AL_H^{\alpha_H} (L_L + BP_t)^{\alpha_L} K_t^{\alpha_K}, \tag{1}$$

where L_H stands for high-skilled workers, L_L are low-skilled workers, P_t are robots, K_t is physical capital (excluding robots), A_t stands for TFP and B is productivity of robots relative to low-skilled workers. Stock of both high-skilled and low-skilled workers is exogenously given. Furthermore, assume constant returns to scale:

$$\alpha_H + \alpha_L + \alpha_K = 1$$

Total output is divided into consumption C_t and savings S_t, investment equal savings $I_t = S_t$. Savings rate s is constant and exogenously given.

Define *total* capital Z_t as sum of non-robot capital K_t and robots P_t, i.e.:

$$Z_t = K_t + P_t$$

Total capital Z_t accumulates according to:

$$\Delta Z_t = I_{t-1} - \delta Z_{t-1} = sY_{t-1} - \delta Z_{t-1}$$

Total capital Z_t can be used either as non-robot capital K_t or as robots P_t, however neither K_t

nor P_t can be negative:

$$0 \leq K_t, 0 \leq P_t$$

Total capital Z_t is divided into K_t and P_t by a social planner in such way that output Y_t is maximized. It is easy to see that since non-robot capital is a necessary input, constraint $0 \leq K_t$ is never binding. However, it is possible to choose zero robots $P_t = 0$ and produce output solely using high-skilled and low-skilled labour and non-robot capital. It is therefore usefull to distinguish between two cases:

3.1 Corner solution: No robots employed

Observe that as K_t approaches zero, marginal product of non-robot capital dY_t/dK_t tends to infinity (standard Inada condition). However, marginal product of robots is not infite if there are no robots employed because robots *are not necessary input* - they can be substituted by low-skilled labour. Therefore, as P_t approaches zero dY_t/dP_t aproaches positive constant.

Therefore, for low levels of Z_t it is optimal to invest only in non-robot capital if for $K_t = Z_t$ and $P_t = 0$ marginal product of non-robot capital exceeds (or is equal to) marginal product of robots, i.e.

$$F_K(L_H, L_L, Z_t, 0) \geq F_P(L_H, L_L, Z_t, 0) \iff \alpha_K \frac{Y_t}{Z_t} \geq B\alpha_L \frac{Y_t}{L_L},$$

where F_K and F_P are partial derivatives with respect to K_t and P_t. Solving for Z yields:

$$Z_t \leq \frac{\alpha_K}{\alpha_L}\frac{L_L}{B} \tag{2}$$

If Z_t satisfies condition (2), production function (1) is reduced to:

$$Y_t = AL_H^{\alpha_H} L_L^{\alpha_L} Z_t^{\alpha_K},$$

Unsurprisingly, if no robots are used in the production, model reduces to simple Solow-like version.

3.2 Inner solution: Robots employed

Robots are employed if $Z_t > \frac{\alpha_K}{\alpha_L}\frac{L_L}{B}$. In this case, marginal product of non-robot and robot capital must be equal:

$$F_K(L_H, L_L, K_t, P_t) \geq F_P(L_H, L_L, K_t, P_t) \iff \alpha_K \frac{Y_t}{K_t} = B\alpha_L \frac{Y_t}{L_L + BP_t}$$

Rearranging produces expression for optimal stock of robots with respect to non-robot capital and unskilled labour:

$$P_t = \frac{\alpha_L}{\alpha_K} K_t - \frac{1}{B} L_L$$

Using this in the production function (1) yields:

$$Y_t = AL_H^{\alpha_H} \left(B \frac{\alpha_L}{\alpha_K} K_t \right)^{\alpha_L} K_t^{\alpha_K}, \tag{3}$$

Now use definition $Z_t = K_t + P_t$ to obtain expression for optimal stock of non-robot capital:

$$K_t = \frac{Z_t + \frac{1}{B} L_L}{1 + \frac{\alpha_L}{\alpha_K}} \tag{4}$$

Insert this into (3) to obtain:

$$Y_t = DAL_H^{\alpha_H} \left(Z_t + \frac{1}{B} L_L \right)^{\alpha_L + \alpha_K}, \tag{5}$$

where $D \equiv \left(B \frac{\alpha_L}{\alpha_K} \right)^{\alpha_L} \left(1 + \frac{\alpha_L}{\alpha_K} \right)^{-\alpha_L - \alpha_H}$

It is straightforward to show that for borderline case $Z_t = \frac{\alpha_K}{\alpha_L}\frac{L_L}{B}$ two versions of production function yields the same value $Y_t = AL_H^{\alpha_H} L_L^{\alpha_L} Z_t^{\alpha_K} = DAL_H^{\alpha_H} \left(Z_t + \frac{1}{B} L_L \right)^{\alpha_L + \alpha_K}$. Also, $\frac{dAL_H^{\alpha_H} L_L^{\alpha_L} Z_t^{\alpha_K}}{dZ_t} = \frac{dDAL_H^{\alpha_H} \left(Z_t + \frac{1}{B} L_L \right)^{\alpha_L + \alpha_K}}{dZ_t}$.

3.3 Combining two cases

Above mentioned analysis enables to write the model in the simple form, where final output is produced by the following production function:

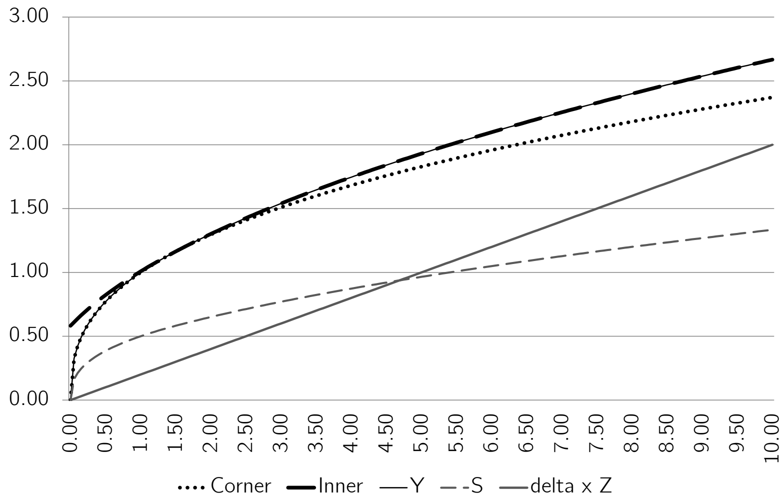

Figure 2: Standard graphical analysis

$$Y_t = \begin{cases} AL_H^{\alpha_H} L_L^{\alpha_L} Z_t^{\alpha_K}, & Z_t \leq \frac{\alpha_K L_L}{\alpha_L B} \\ DAL_H^{\alpha_H} \left(Z_t + \frac{1}{B} L_L\right)^{\alpha_L + \alpha_K}, & Z_t > \frac{\alpha_K L_L}{\alpha_L B} \end{cases}$$

Total capital Z_t accumulates according to:

$$\Delta Z_t = I_{t-1} - \delta Z_{t-1} = sY_{t-1} - \delta Z_{t-1}$$

Figure 2 gives standard graphical analysis for the following calibration: $\alpha_H = 4/8$, $\alpha_L = 1/8$, $\alpha_K = 3/8$, $A = 1$, $B = 2$, $s = 0.5$ and $\delta = 0.2$. Borderline value of Z_t when robots begin to be employed in the production process is in this calibration $Z_t = \frac{\alpha_K}{\alpha_L} \frac{L_L}{B} = 1.5$.

Dotted line corresponds to corner solution. For Z_t exceeding borderline value of 1.5, output is given by the inner solution. Observe that the inner and the corner solution differs in *the elasticity of output with respect to* Z_t. Whereas for low values of Z_t, elasticity is α_K, for high values of Z_t, elasticity is $\alpha_L + \alpha_K$. Also, observe that for high values of Z_t, actual output is higher than what would be the case if no robots were used - introduction of robots alleviates tendency to decreasing marginal product, but is not enough to completely overcome decreasing returns to produced factors.

3.4 Steady-state analysis I.: No robots employed

Steady state is reached if $\Delta Z_t = 0$, that is:

$$Z^* = \frac{sY^*}{\delta}$$

Once again, two cases are possible. Either productivity of robots relative to low-skilled workers is so low that even in the steady state it is not optimal to use robots ($Z^* < \frac{\alpha_K L_L}{\alpha_L B}$). In this case model reduces to standard neoclassical model with well-known behaviour, steady state being given by:

$$Z^* = \left(\frac{sAL_H^{\alpha_H} L_L^{\alpha_L}}{\delta}\right)^{\frac{1}{1-\alpha_K}} \tag{6}$$

Combining expression $\frac{\alpha_K}{\alpha_L}\frac{L_L}{B}$ for borderline value of Z, with (6) reveals that this steady state applies if:

$$B \leq \frac{\alpha_K}{\alpha_L}\left(\frac{\delta}{sA}\right)^{\frac{1}{1-\alpha_K}}\left(\frac{L_L}{L_H}\right)^{\frac{\alpha_H}{1-\alpha_K}} \tag{7}$$

Robots are not employed in the steady state if low B is combined with high depreciation rate, low TFP, low savings rate and high ratio of low-skilled to high-skilled labour.

Steady state output can be written as:

$$Y^* = \left(\frac{s}{\delta}\right)^{\frac{\alpha_K}{1-\alpha_K}}(AL_H^{\alpha_H} L_H^{\alpha_L})^{\frac{1}{1-\alpha_K}} \tag{8}$$

Assuming competitive markets, prices of factors are easy to derive:

- Wages of high-skilled workers are given by (F_{L_H} being partial derivative with respect to L_H):

$$w_H^* = F_{L_H}(L_H, L_L, K^*, 0) = \alpha_H A^{\frac{1}{1-\alpha_K}}\left(\frac{s}{\delta}\right)^{\frac{\alpha_K}{1-\alpha_K}}\left(\frac{L_L}{L_H}\right)^{\frac{\alpha_L}{1-\alpha_K}} \tag{9}$$

- Analogically, wages of low-skilled workers are given by (F_{L_L} being partial derivative with respect to L_L):

$$w_L^* = F_{L_H}(L_H, L_L, K^*, 0) = \alpha_L A^{\frac{1}{1-\alpha_K}} \left(\frac{s}{\delta}\right)^{\frac{\alpha_K}{1-\alpha_K}} \left(\frac{L_H}{L_L}\right)^{\frac{\alpha_H}{1-\alpha_K}} \tag{10}$$

- Returns to non-robot capital are given by:

$$r_K^* = F_K(L_H, L_L, K^*, 0) = \alpha_K \frac{\delta}{s} \tag{11}$$

- Income shares of high-skilled labour, low-skilled labour, non-robot physical capital and physical capital on output are α_H, α_L, α_K and 0 respectively.

Observe that wages of both skilled and unskilled workers are a positive function of overall productivity A. Also, elasticity of both wages with respect to A is the same - $\frac{1}{1-\alpha_K}$. Increases in TFP produces same relative changes wages for both skilled and unskilled workers.

On the other hand, returns to non-robot capital do not depend on A. The reason is easy to understand - higher A increases marginal product of K but also enables accumulation of non-robot capital what produces counteracting effect.

Consumption-maximizing savings rate s_{gold} can be obtain by differentiating $Y^*(1-s)$ (Y^* given by (8)) with respect to s and putting it equal to zero. This yields (akin to Solow model) $s_{gold} = \alpha_K$.

3.5 Steady-state analysis II.: Robots employed

Let us focus on the second case, when use of robots is profitable in the steady state, i.e. if condition (7) is not satisfied. In this case, the inner solution applies and $Y_t = DAL_H^{\alpha_H}\left(Z_t + \frac{1}{B}L_L\right)^{\alpha_L+\alpha_K}$, therefore:

$$Z^* = \frac{sDAL_H^{\alpha_H}\left(Z^* + \frac{1}{B}L_L\right)^{\alpha_L+\alpha_K}}{\delta} \tag{12}$$

Closed-form solution of (12) does not exist. However, characteristics of steady state can be inferred by assuming that B is sufficiently large what makes $\frac{1}{B}L_L$ negligible. This enables to write:

$$Z^* \approx \frac{sDL_H^{\alpha_H} A\left(Z^*\right)^{\alpha_L+\alpha_K}}{\delta}$$

Steady state value of Z is given by:

$$Z^* \approx \left(\frac{sDAL_H^{\alpha_H}}{\delta}\right)^{\frac{1}{1-\alpha_L-\alpha_K}} \tag{13}$$

To obtain expression for steady-state value of output, plug (13) into (5) and once again use assumption that B is large enough what makes $\frac{1}{B}L_L$ negligible. This enables to write steady-state output as:

$$Y^* \approx \left(\frac{s}{\delta}\right)^{\frac{\alpha_L+\alpha_K}{1-\alpha_L-\alpha_K}} \left(DAL_H^{\alpha_H}\right)^{\frac{1}{1-\alpha_L-\alpha_K}} \tag{14}$$

By taking derivative of output as defined by (1) with respect to L_H, L_L, K and P and using (13), wages and returns to both non-robot and robot capital can be obtained.

- Wages of high-skilled workers are given by:

$$w_H^* = F_{L_H}(L_H, L_L, K^*, P^*) \approx \alpha_H \left(DA\right)^{\frac{1}{1-\alpha_L-\alpha_K}} \left(\frac{s}{\delta}\right)^{\frac{\alpha_L+\alpha_K}{1-\alpha_L-\alpha_K}} \tag{15}$$

- To express return of physical capital, use, expression for steady state Z^* (equation (13)), expression for optimal non-robot capital (equation (4)) and use assumption that $\frac{1}{B}L_L$ is negligible. This produces:

$$r_K^* = F_K(L_H, L_L, K^*, P^*) \approx \alpha_K \frac{\delta}{s}$$

- Recall that marginal products of non-robot and robot capital must be equal, therefore:

$$r_P^* = F_P(L_H, L_L, K^*, P^*) = r_K^* \approx \alpha_K \frac{\delta}{s}$$

- To determine wage of non-skilled workers observe that based on (1), $\frac{dY_t}{dL_L} = \frac{1}{B}\frac{dY_t}{dP_t}$. Therefore:

$$w_L^* = F_{L_L}(L_H, L_L, K^*, P^*) = \frac{r_P^*}{B} = \frac{r_K^*}{B} \approx \frac{\alpha_K}{B}\frac{\delta}{s} \tag{16}$$

- Income shares of high-skilled labour is α_H, income share of non-robot capital is α_K, share wages of low-skilled workers on total output is given by $\alpha_L \frac{L_L}{L_L+BP^*}$, share of returns to robot physical capital is equal to $\alpha_L \frac{BP_t}{L_L+BP^*}$. Since $P* > 0$, income share of low-skilled workers is lower than is the case with no robots employed.

It is interesting to contrast this results with situation where productivity of robots is too low to motivate use of the automation technology. Whereas in this case *both* wages of high-skilled and low-skilled workers were positively linked to overall productivity A, in this case *only wages of high-skilled workers are positive function of productivity.* Furthermore, their elasticity with respect to A increases to $\frac{1}{1-\alpha_K-\alpha_L}$. If robots are used in the production process, high-skilled worker benefit even more from the increase in TFP because higher TFP leads to higher stock of robots what increases their productivity.

On the other hand, competition of robots always drives wages of low-skilled workers down to $w_L^* \approx \frac{\alpha_K}{B}\frac{\delta}{s}$ not depending on A. Crucial force behind this is the fact that robots which compete with low-skilled workers *are accumulated as TFP increases.*

However, recall that this results applies for large B. Therefore, Table 1 gives steady-state analysis for intermediate values of B. Numerical solutions are produced for two levels of TFP - $A = 1.0$ and $A = 1.5$. Other parameters are calibrated as above, i.e. $\alpha_H = 4/8$, $\alpha_L = 1/8$, $\alpha_K = 3/8$, $s = 0.5$ and $\delta = 0.2$. Three different B's are assumed.

Table 1: Numerical analysis of impact of changes in TFP on incomes and income distribution: Steady state

case		w_H	w_L	r_K	MP of P	share L_H	share L_L	share K	share P
$B = 0.01$	A=1.0	0.866	0.217	0.150	0.002	0.500	0.125	0.375	0.000
	A=1.5	1.658	0.414	0.150	0.004	0.500	0.125	0.375	0.000
	Index	1.913	1.913	1.000	1.913	1.000	1.000	1.000	-
$B = 1.00$	A=1.0	0.875	0.163	0.163	0.163	0.500	0.093	0.375	0.032
	A=1.5	1.783	0.180	0.180	0.180	0.500	0.050	0.375	0.075
	Index	2.037	1.105	1.105	1.105	1.000	0.542	1.000	2.331
$B = 2.00$	A=1.0	0.937	0.090	0.181	0.181	0.500	0.048	0.375	0.077
	A=1.5	2.001	0.095	0.190	0.190	0.500	0.024	0.375	0.101
	Index	2.135	1.054	1.054	1.054	1.000	0.494	1.000	1.318

For low $B = 0.01$ no robots are employed (marginal product of robots is smaller than returns to capital r_K). Increase in A produces *the same* increase in wages of both high-skilled and low-skilled workers (row denoted 'Index' shows relative changes produces by increase of A from $A = 1.0$ to $A = 1.5$). Both wages increase by factor 1.913. As mentioned, from equations (9) and (10) it follows that elasticity of both with respect to A is $\frac{1}{1-\alpha_K}$, $1.5^{\frac{1}{1-3/8}} = 1.913$.

Compare this to the situation with intermediate productivity of robots $B = 1$. Higher TFP increases wages of both high-skilled and low-skilled workers but wage increase produced enjoyed by high-skilled workers is much higher (2.037-fold) than that of low-skilled workers (1.105-fold).

This contrast is even more pronounced for $B = 2.00$. As shown above, for B large enough, wages of low skilled workers do not depend on TFP. On the other hand, elasticity of wages of high-skilled workers with respect to TFP is $\frac{1}{1-\alpha_K-\alpha_L}$. In this calibration, 1.5-fold increase in A would increase steady-state value of w_H by $1.5^{\frac{1}{1-1/8-3/8}} = 2.25$.

Furthermore, Table 1 provides information about the effect of increase in robot productivity B

holding TFP constant. Observe that since there is certain degree of complementarity between high-skilled workers and robots, their wages increase. On the other hand, since low-skilled labour and robots are strict complements, low-skilled workers suffer drop in wages.

Finally, to obtain consumption-maximizing savings rate s_{gold}, differentiate $Y^*(1-s)$ (Y^* given by (14)) with respect to s and put it equal to zero. This produces $s_{gold} = \alpha_L + \alpha_K$. Note that once robots are effectively employed in the production process, s_{gold} increases from α_K to $\alpha_L + \alpha_K$ because elasticity of steady-state output with respect to *all* produced capital (both robot and non robot) is higher than elasticity with respect to non-robot capital only.

3.6 Transitional dynamics

To illustrate the effect of accumulation of robots we simulate transitional dynamics of the model using above calibration ($A = 1$, $B = 2$). Starting value $Z_0 = 0.5$ is used.

Figure 3 gives accummulation of total capital Z_t, non-robot capital K_t and robots P_t (time on horizontal axis) and output Y_t. Observe that robots begin to being accumulated only after stock of non-robot capital reaches certain level.

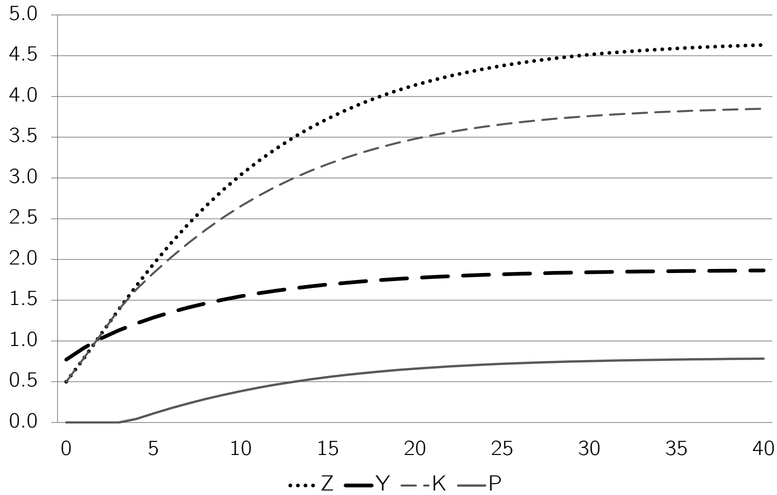

Figure 3: Transitional dynamics I.: Fundamental variables

Figure 4 gives prices of factors of production. As non-robot capital are being accumulated, its marginal product decreases. On the other hand, since K and P complement each other in production function, marginal product of robots (denoted MP of P) increases. *Robots began to be accumulated at the moment when their marginal product is equal to that of non-robot capital.* Observe that this is precisely the moment when wages of non-skilled worker starts to *decrease.*

This is due to competition on part of robots. Observe that since $B = 2$, wages of non-skilled workers are half of marginal product of robots. Whereas low-skilled workers are hurt by robot competition, high-skilled workers enjoy sustained increase in wages (until economy converges to the steady state).

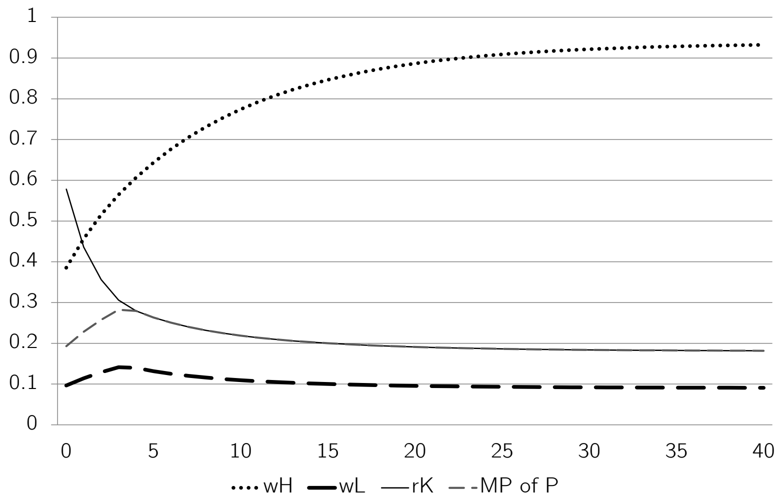

Figure 4: Transitional dynamics II.: Wages and returns to non-robot capital

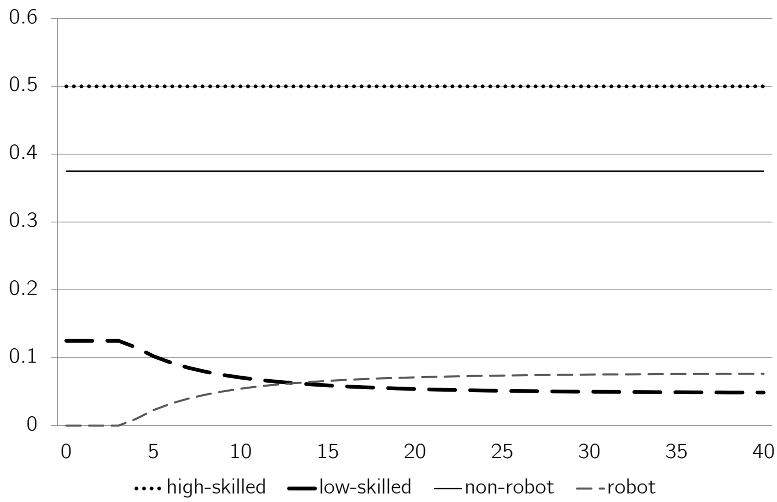

Figure 5: Transitional dynamics III.: Income shares

This dynamics has implications for income shares. Whereas income share of high-skilled workers and non-robot capital is constant, accumulation of robots leads to shift in income shares from low-skilled workers to robots. At certain point, share of robots (or rather their owners) on total income can overtake share of low-skilled workers.

Finally, Figure (6) shows the effect of increase in TFP on prices of factors of production. During the first five periods ($t = 0, ..., 4$), economy is in the steady state and $A = 1$. In period $t = 5$, TFP shifts to $A = 1.5$. Observe that both wages of low-skilled and high-skilled workers increases suddenly. However, as economy converges to the new steady state, wages of high-skilled workers tend to increase due to accumulation of both non-robot and robot capital. On the other hand, wages of low-skilled worker tend to decrease due to competition of newly created robots. In the long run, effect is very small and in case of large B negligible (recall that for large B, wages of low-skilled workers in steady states do not depend on A).

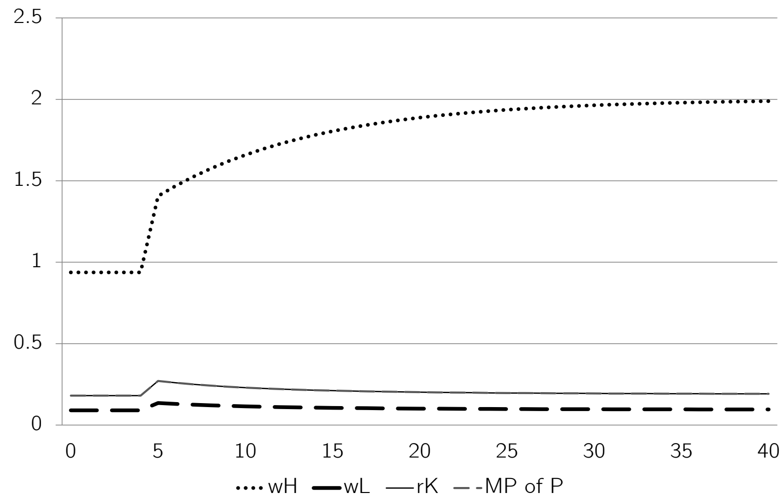

Figure 6: Transitional dynamics IV.: Effects of increase in TFP on prices of factors of production

4 A model with population growth and technological progress

One of the most important results of previous analysis is that in case of sufficiently high B, wages of low-skilled workers do not depend on level of TFP. In this section we show that this results generalizes to environment with population growth and exogenous technological progress. We show that **wages of low-skilled labour are stagnant despite exogenous technological progress**.

Assume rate of population growth n and rate of technological progress g. Production function takes the following form:

$$Y_t = A\left[(1+g)^t(1+n)^tL_{H,0}\right]^{\alpha_H}\left[(1+g)^t(1+n)^tL_{L,0}+(1+g)^tBP_t\right]^{\alpha_L}K_t^{\alpha_K}, \qquad (17)$$

where $L_{H,0}$ and $L_{L,0}$ are numbers of high-skilled and low-skilled workers in time $t = 0$. Total number of workers in time t is equal to $(1+n)^t L_{H,0}$ and $(1+n)^t L_{L,0}$.

Capital is accumulated as before, i.e.:

$$\Delta Z_t = I_{t-1} - \delta Z_{t-1} = sY_{t-1} - \delta Z_{t-1}$$

$$Z_t = K_t + P_t$$

$$0 \leq K_t, 0 \leq P_t$$

Following the same reasoning, robots are not employed if marginal product of robots is lower than marginal product of non-robot capital even if $P_t = 0$. Condition (2) changes into:

$$Z_t \leq \frac{\alpha_K}{\alpha_L} \frac{(1+n)^t L_{L,0}}{B} \tag{18}$$

In this case, **corner solution** is used and production function reduces to:

$$Y_t = A\left[(1+g)^t (1+n)^t L_{H,0}\right]^{\alpha_H} \left[(1+g)^t (1+n)^t L_{L,0}\right]^{\alpha_L} Z_t^{\alpha_K},$$

Now assume without loss of generality $L_{H,0} + L_{L,0} = 1$ and define variables *per effective worker* as $\hat{x}_t = \frac{X_t}{(1+g)^t (1+n)^t}$. Model can be written as:

$$y_t = AL_{H,0}^{\alpha_H} L_{L,0}^{\alpha_L} \hat{z}_t^{\alpha_K},$$

$$\Delta \hat{z}_t = \frac{s\hat{y}_{t-1} - (\delta + g + n)\hat{z}_{t-1}}{(1+g)(1+n)} \tag{19}$$

When no robots are employed, model behaves *as if* converging to steady state when $\Delta \hat{z}_t = 0$, i.e.:

$$\hat{z}_{PSS}^* = \left(\frac{sAL_{H,0}^{\alpha_H} L_{L,0}^{\alpha_L}}{\delta + g + n}\right)^{\frac{1}{1-\alpha_K}} \tag{20}$$

We will show that steady state defined by (20) is *not a true steady state*, we will refer to it as *pseudo-steady state* (this is what PSS in $\hat{z}_{PSS}^*$ stands for). Assume that economy converged to $\hat{z}_{PSS}^*$, growth rate of Z_t is therefore $g + n$ (ignoring second-order terms). Therefore, no matter how low B is, after sufficient time, condition (18) cease to hold, robots start to be employed in the production process and (20) is no longer applicable. However, it is important to stress that for low B, economy can for a relatively long time behave as if converging to $\hat{z}^*$ and growth rate of economy can for a considerable period be close to $g+n$ (growth rate of wages of both low-skilled and high-skilled workers as well as of output and consumption per capita is close to g; returns to non-robot capital $r_{K,t}$ converge to pseudo-steady state value $\alpha_K \frac{\delta+g+n}{s}$). Dynamics of the economy changes only after robots begin to be employed in the production and we will show that growth rate will eventually converge to value higher than $g+n$.

If (18) is not satisfied, **inner solution applies**, and balancing marginal products of robot and non-robot capital requires $P_t = \frac{\alpha_L}{\alpha_K} K_t - \frac{(1+n)^t}{B} L_{L,0}$. Following the same steps as in the section 3.2 yields:

$$Y_t = DA\left[(1+g)^t\right]^{\alpha_L}\left[(1+g)^t(1+n)^t L_{H,0}\right]^{\alpha_H}\left[Z_t + \frac{(1+n)^t}{B} L_{L,0}\right]^{\alpha_L+\alpha_K},$$

where D is defined as before. Once again expressing variables in per-effective-worker terms enables to write:

$$\hat{y}_t = DA\left[(1+g)^t\right]^{\alpha_L} L_{H,0}^{\alpha_H}\left[\hat{z}_t + \frac{1}{B}\frac{L_{L,0}}{(1+g)^t}\right]^{\alpha_L+\alpha_K}$$

Use assumption that B is sufficiently low and/or t sufficiently high what makes $\frac{1}{B}\frac{L_{L,0}}{(1+g)^t}$ is negligible (this is always true in the steady state as t grows to infinity). Production function can be written as:

$$\hat{y}_t = DA\left[(1+g)^t\right]^{\alpha_L} L_{H,0}^{\alpha_H}\hat{z}_t^{\alpha_L+\alpha_K} \tag{21}$$

Law of motion of $\hat{z}$ is given by (19).

Model is once again reduced to well-known Solow form. To solve the model, perform one more transformation of variables, $\tilde{x}_t = \hat{x}_t(1+g)^{-\frac{\alpha_L}{1-\alpha_L-\alpha_K}t}$). Steady state growth rate of both $\hat{y}_t$ and $\hat{z}_t$ is equal to $1-(1+g)^{\frac{\alpha_L}{1-\alpha_L-\alpha_K}} \approx g\frac{\alpha_L}{1-\alpha_L-\alpha_K}$. Therefore, Y_t and Z_t grow at the rate:

$$g_Y^* = g_Z^* \approx g\frac{1-\alpha_K}{1-\alpha_L-\alpha_K} + n = \tilde{g} + n,$$

where $\tilde{g} \equiv g\frac{1-\alpha_K}{1-\alpha_L-\alpha_K}$. Since $K_t = \frac{Z_t + \frac{(1+n)^t}{B}L_{L,0}}{1+\frac{\alpha_L}{\alpha_K}}$ (analogically to (4)) and $P_t = \frac{\alpha_L}{\alpha_K}K_t - \frac{(1+n)^t}{B}L_{L,0}$, growth rate of both non-robot and robot capital approaches g_Y^*.

It is interesting to contrast this result with standard Solow model where long-term growth rates of output and capital converge to g. In standard model, technological progress increases effective amount of workers, i.e. of factor of production which is *not produced* (hence *labour-augmenting* technological progress). In model with industrial robots, improvements in technology also increases affective number of robots - produced factor of production. The greater α_L, the more important robots are and the higher is growth premium. As α_L converges to 0, growth rate converges to standard $g + n$.

Growth-rates of prices of factors of production can now be easily derived. Before providing analytical solution, let us not that Cobb-Dougles structure of the production function ensures that income share of high-skilled labour is constant. Since growth rate of skilled labour is equal to n, **wages of high-skilled workers** exhibit steady-state growth rate $g^*_{w_H} = g^*_Y - n \approx \tilde{g}$.

Furthermore, income share of capital is also constant and since growth rates of K_t and Y_t are equal, **returns to non-robot capital** in the steady state are constant. Therefore, **returns to robot capital** are also constant. Furthermore, since marginal product of non-skilled labour is proportional to marginal product of robots ($\frac{dY_t}{d(1+n)^t L_{L,0}} = \frac{1}{B}\frac{dY_t}{dP_t}$), **wages of low-skilled workers** are also constant.

However, even though there is no *growth* effect of g on wages of low-skilled workers, it can be shown that there is a *level* effect. Observe that model described by equations (21) and (19) converge to:

$$\tilde{z}^* \approx \left(\frac{sDAL_{H,0}^{\alpha_H}}{\delta + \tilde{g} + n} \right)^{\frac{1}{1-\alpha_L-\alpha_K}} \tag{22}$$

Total capital in the steady state can be written as:

$$Z^*_t = \tilde{z}^* (1+g)^{\frac{1-\alpha_K}{1-\alpha_L-\alpha_K}t} (1+n)^t \tag{23}$$

Higher g have negative level effect on Z^*_t and K^*_t which in turn increases marginal product of non-robot capital.

To obtain analytical solutions for prices of factors of production use (22), (23) and once again assumption that $\frac{(1+n)^t}{B} L_{L,0}$ is negligible.

- Differentiate (17) with respect to $(1+n)^t L_{H,0}$ to obtain wage of high-skilled workers:

$$w^*_{H,t} \approx \alpha_H (DA)^{\frac{1}{1-\alpha_L-\alpha_K}} \left(\frac{s}{\delta + \tilde{g} + n} \right)^{\frac{\alpha_L+\alpha_K}{1-\alpha_L-\alpha_K}} (1+g)^{\frac{1-\alpha_K}{1-\alpha_L-\alpha_K}t}$$

- Return to non-robot capital is obtained by differentiating (17) with respect to K:

$$r_K^* \approx \alpha_K \frac{\delta + \tilde{g} + n}{s}$$

- Marginal products of robot and non-robot capital are equal:

$$r_P^* = r_K^* \approx \alpha_K \frac{\delta + \tilde{g} + n}{s}$$

- Marginal product of low-skilled workers (and their wage) is proportional to marginal product of robots:

$$w_L^* = \frac{r_P^*}{B} = \frac{r_K^*}{B} \approx \frac{\alpha_K}{B} \frac{\delta + \tilde{g} + n}{s}$$

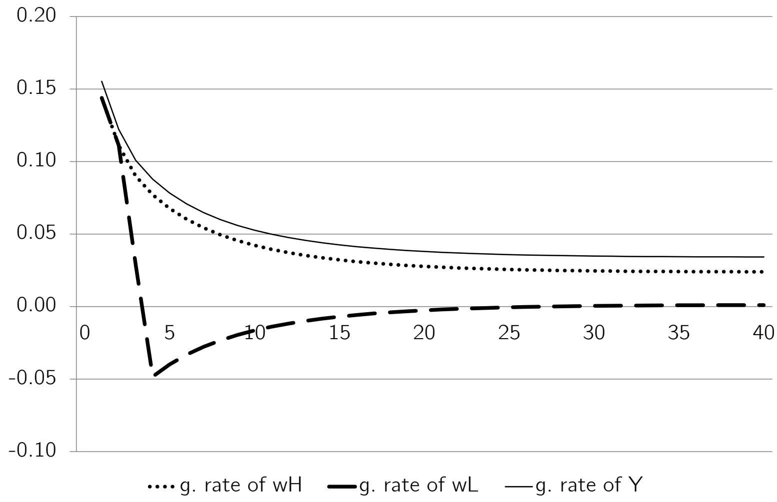

Figure 7: Trans. dynamics in model with population growth and technological progress I.

Figure 7 gives growths rates of wages of high-skilled workers, low-skilled workers and output along the transition path. Same calibration is used as before ($B = 2$) and $g = 0.02$ and $n = 0.01$. Starting value $Z_0 = 0.5$ is used. During the periods $t = 1, 2, 3$ growth rate wages of both type of workers is the same. Growth rate of output is slightly higher since it is also driven by population growth. In period $t = 4$ robots are introduced what causes *drop* in wages of low-skilled worker. Growth rate remains negative until model reaches steady state where $g_{w_L} = 0$, $g_{w_H} \approx g\frac{1-\alpha_K}{1-\alpha_L-\alpha_K} = 0.025$ and $g_Y \approx g\frac{1-\alpha_K}{1-\alpha_L-\alpha_K} + n = 0.035$. However, introduction of robots into production process does not necessarily produce negative growth rate of w_L, simple reduction of growth rate is also a possible result (as shown below).

To provide more information about the transition until robots are used, Figure 8 depicts transition with lower B. Value $B = 0.5$ is used. Robots begin to be used in period $t = 29$. Until

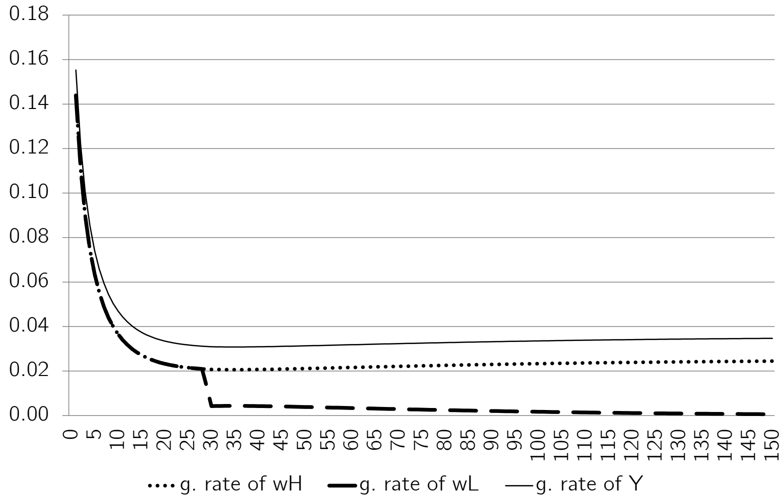

Figure 8: Trans. dynamics in model with population growth and technological progress II.

then, economy has Solow-like features - growth rates of high-skilled and low-skilled labour are the same and they converge to the rate of technological progress $g = 0.02$. Growth rate of output approaches $g + n = 0.03$. However, once robots begin to be used, growth rate of wages of low-skilled workers decreases, even if it does not drop below zero as was the case with $B = 2$ (Figure 7). On the other hand, growth rate of wages of high-skilled workers begin to increase and gradually approaches $g_{w_H} \approx g\frac{1-\alpha_K}{1-\alpha_L-\alpha_K} = 0.025$. Growth rate of output also increases and approaches $g_Y \approx g\frac{1-\alpha_K}{1-\alpha_L-\alpha_K} + n = 0.035$.

Dynamics depicted in Figure 8 suggests that it is possible for a richer economy to grow faster than a poorer economy. This creates a possibility of *divergent* development. To illustrate how automation can lead to economic divergence can come abou,t we perform two simulations assuming two developed economy R and developing economy P. We assume that in economy R, initial stock of capital is higher than in economy P, $Z_0^R > Z_0^P$, in particular $Z_0^R = 3.5$ and $Z_0^R = 2.9$. We also assume that *total factor productivity is higher in developed country R than in developing country P, i.e.* $A_0^R > A_0^P$. Values of all parameters ($B = 0.8$, $s = 0.5$, $\delta = 0.2$, $\alpha_H = 4/8$. $\alpha_L = 1/8$, $\alpha_K = 3/8$, $g = 0.02$ and $n = 0.01$) are same in both economies.

In **simulation 1**, we assume that *total factor productivity A is constant in both countries*, $A^R = 1$ whereas $A^P = 0.9$. Figure 9 depicts σ-convergence of log of output per capita for the two countries ('sigma 1'). Observe that initially σ decreases since differences in initial Z between two countries are relatively high and differences in A are relatively low (poorer economy P is further from its steady state than richer economy R). If there had been no possibility to employ robots in the production process, σ would have been gradually converging to $\sigma = 0.119$ (see line 'sigma 0 (no robots)' in Figure 9). However, in period $t = 4$ richer country R begins to employ robots in the production and begin to enjoy robot-related growth premium. Poorer country P follows suit in period $t = 13$, however, beginning $t = 10$, richer country already enjoys higher economic growth. This creates divergence in log of output per capita. Coefficient of σ-convergence begins to increase and gradually converges to $\sigma = 0.149$.

To better understand causes of economic divergence, recall that in steady state (or pseudo steady state) elasticity of output with respect to all economic fundamentals such as s, A, δ, g and n is *higher* if robots are used (compare 20 and 22). Once robots are employed in the production process, differences in fundamentals are reflected in *higher differences* our output per capita.

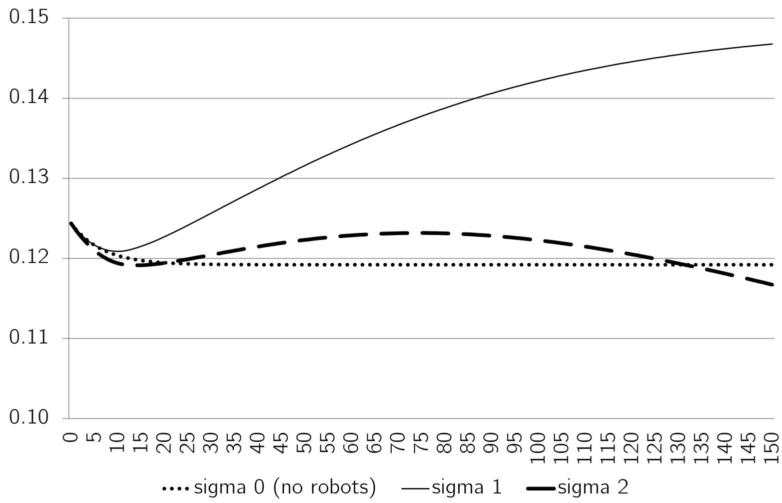

Figure 9: Non-linear development of σ-convergence

In **simulation 2**, we assume that even though total factor productivity in developing economy P is lower than in developed economy R, *there is a slow convergence in TFP*. In particular, we assume $A_t^R = 1$ for all t, $A_0^P = 0.9$ and $A_t^P = 0.9985A_{t-1}^P + 0.0015A_t^R$ what implies rate of TFP convergence of 0.15%. Development of σ-convergence is once again given in Figure 9 ('sigma 2').

During the initial phase, σ decreases due to standard Solow-like convergence effect (due to our calibration, poorer economy P is further from its steady state than richer economy R) as well as due to convergence of TFP (therefore, convergence is faster that in simulation 1). Once robots are employed in the production process, richer country R starts to enjoy higher economic growth due to robot premium. In other words, use of robots increases the elasticity of steady-state output with respect to fundamentals and causes divergence of output per capita. However, after sufficient time, convergence in TFP enables two economies to converge and σ converges to $\sigma = 0$.

Note that the main driver of divergence is higher elasticity of steady-state output with respect two fundamentals. Unless two countries differ in fundamentals, robot technology is not sufficient to generate divergence.

5 Empirical estimation

To obtain estimable equations, first assume that between time t_0 and time t_1, robots are not employed in the production process. Economy therefore converges to the pseudo-steady state defined by (20). Use lower case symbols for variables in per capita terms (i.e. $x_t = X_t/(1+n)^t$) and write output per capita in pseudo-steady state as:

$$\begin{aligned}\log y^*_{PSS,t} =& \frac{1}{1-\alpha_K}\log A + t\log(1+g) + \frac{\alpha_K}{1-\alpha_K}\log s + \frac{\alpha_H}{1-\alpha_K}\log L_{H,0} + ...\\ &...\frac{\alpha_L}{1-\alpha_K}\log L_{L,0} - \frac{\alpha_K}{1-\alpha_K}\log(\delta+g+n)\end{aligned} \tag{24}$$

Assuming rate of convergence to steady state λ_{PSS}, growth rate between t_0 and t_1 can be written as:

$$\log y_{t_1} - \log y_{t_0} = \left[1-(1-\lambda_{PSS})^{t_1-t_0}\right]\log y^*_{PSS,t} - \left[1-(1-\lambda_{PSS})^{t_1-t_0}\right]\log y_{t_0} \tag{25}$$

Now assume that in time t_0 robots begin to be employed. Economy converges to the true steady state (22). Therefore:

$$\begin{aligned}\log y^*_t =& \frac{1}{1-\alpha_L-\alpha_K}\log DA + t\frac{1-\alpha_K}{1-\alpha_L-\alpha_K}\log(1+g) + \frac{\alpha_L+\alpha_K}{1-\alpha_L-\alpha_K}\log s + ...\\ &... + \frac{\alpha_H}{1-\alpha_L-\alpha_K}\log L_{H,0} - \frac{\alpha_L+\alpha_K}{1-\alpha_L-\alpha_K}\log(\delta+\tilde{g}+n)\end{aligned} \tag{26}$$

Growth rate between t_0 and t_1 is given by:

$$\log y_{t_1} - \log y_{t_0} = \left[1-(1-\lambda)^{t_1-t_0}\right]\log y^*_t - \left[1-(1-\lambda)^{t_1-t_0}\right]\log y_0, \tag{27}$$

where λ is rate of convergence to the true steady state.

It is important to stress that rate of convergence to the pseudo-steady state λ_{PSS} differs from the rate of convergence to the true steady state λ. This is due to the fact that in the neighbourhood of the steady state, speed of convergence is determined by (i) elasticity of production with respect to Z_t (denote it by α) and (ii) the rate of effective depreciation (denote it by δ_{eff}). In particular, speed of convergence is given by $(1-\alpha)\delta_{EFF}$ (see for example Barro - Sala-i-Martin (2004), chapter 1.2.13). In case that no robots are employed, elasticity of production with respect to Z_t is $\alpha = \alpha_K$ and effective depreciation is $\delta_{EFF} = n+g+\delta$. In the true steady state robots are employed, elasticity of production with respect to Z_t is $\alpha = \alpha_K + \alpha_L$ and effective depreciation is $\delta_{EFF} = n + \tilde{g} + \delta$. Therefore:

$$\lambda_{PSS} \approx (1-\alpha_K)(n+g+\delta)$$

$$\lambda \approx (1-\alpha_K-\alpha_L)(n+\tilde{g}+\delta)$$

For most reasonable values of parameters $\lambda_{PSS} > \lambda$ meaning that rate of conditional convergence is higher for economies in which robots are not employed.

Natural way of testing the model on panel data is therefore estimating a following regression (CS and IS standing for corner and inner solution respectively):

$$\log y_t = \begin{cases} \begin{matrix} \beta_0^{CS} + \beta_1^{CS}\log s_{i,t} + \beta_2^{CS}\log L_{H,i,t} + \beta_3^{CS}\log L_{L,i,t} + ... \\ ... + \beta_4^{CS}\log(\delta+g+n) + \beta_5^{CS}\log y_{i,t-1} + \mu_i^{CS} + \rho_t^{CS} + \epsilon_{i,t} \end{matrix}, & \text{for } P_{i,t-1} = 0 \\ \begin{matrix} \beta_0^{IS} + \beta_1^{IS}\log s_{i,t} + \beta_2^{IS}\log L_{H,i,t} + \beta_3^{IS}\log L_{L,i,t} + ... \\ ... + \beta_4^{IS}\log(\delta+\tilde{g}+n) + \beta_5^{IS}\log y_{i,t-1} + \mu_i^{IS} + \rho_t^{IS} + \epsilon_{i,t} \end{matrix}, & \text{for } P_{i,t-1} > 0 \end{cases}$$

Coefficients from the regression are easily mapped into terms in the equation (24)-(27). For example, $\beta_1^{CS} = \left[1-(1-\lambda_{PSS})^{t_1-t_0}\right]\frac{\alpha_L+\alpha_K}{1-\alpha_L-\alpha_K}$, $\beta_5^{CS} = \left[1-(1-\lambda_{PSS})^{t_1-t_0}\right]$, $\beta_3^{IS} = 0$, $\beta_2^{IS} = 1$ and so on. This allows us to test several restrictions on the parameters and derive the values of parameters in the original model. In the following empirical analysis, we focus on two predictions of the model. First, the speed of convergence towards a steady state should be higher for economies without robots and thus the coefficient for a lagged income per capita β_5 should be lower for non-robot countries. Second, he long-run elasticity of income per capita with respect to saving rate $\beta_1/(1-\beta_5)$ should be higher in countries with robots (delta method is used to calculate the standard deviation of the transformed coefficients).

5.1 Data

Data on income per capita, economic growth, population and savings are obtained from Penn World Tables 9.0. We use the database of International Federation for Robotics (IFR) for the stock of robots employed by countries. Other covariates are taken from World Bank´s World Development Indicators database. Our baseline estimation covers 1993 - 2014 time period and 65 countries. These are the countries that in some point in time adopted robots in the production process. In Appendix, we present the estimation for the whole sample of countries available (111 countries).

In a line with Mankiw-Romer-Weil (1992), we droped countries with population less than 1 million of inhabitants and oil-producing countries [3]. We assume that the efficient depreciation of capital per worker equals $(n+g+\delta) = n+0.050$ for non-robot countries and equals $(n+\tilde{g}+\delta) = (n+0.055)$ for countries that produce by utilizing robots in the production process.

5.2 First empirical results

Our model predicts different speed of convergence towards steady state for countries which do not use robots in the production and countries which employ automation technology. We test this hypothesis by running regressions for two separate samples - countries without robots ('no robots' sample) and countries with robots ('robots' sample). We split the sample based on whether robots were used in production in period $t-1$.

First, we estimate the parameters by simple OLS. However, in dynamic panels with fixed country-specific effects Nickel bias tends to bias estimates of speed of convergence as well as other parameters (Nickell, 1981). Therefore, we also perform GMM-estimation using Allerano-Bond estimator (Arellano and Bond, 1991) (we restrict maximal number of lags of dependent variables used as instruments to 3).

Table 2 gives estimation results. The coefficient for lagged income per capita is positive and significant for both samples and estimation methods. Compared to 'robot sample', the coefficient is *smaller* in the sample of countries producing without automation technology. Significant differences between these two coefficients (0.695 vs 0.910 from OLS estimation, and 0.419 and 0.818 from Allerano-Bond estimation; confidence intervals do not overlap) confirm the hypothesis of *higher* conditional convergence towards steady state for no-robots countries.

The long-run elasticity of income per capita with respect to saving rate (calculated as $\frac{\beta_1^{CS}}{1-\beta_5^{CS}}$ and $\frac{\beta_1^{IS}}{1-\beta_5^{IS}}$) is higher for 'robot' sample (0.18 from OLS estimates and 0.04 from Allerano-Bond estimates) than in 'no robot' sample (0.49 and 0.44 respectively).

The simple rule for a sample split suggested above seems to be too sharp. For example, in 2014 Pakistan was using 2 industrial robots, therefore, it is included in 'robots sample'. However, do two industrial robots in the country of 185 million people signify beginning of the transition to automation technology? Most likely not. Therefore, we spilt the sample based on non-zero threshold for number fo robots per population of 1 million. We perform estimations moving the threshold from 0 to 10 robots per 1 million.

[3] United Arab Emirates, Bahrain, Gabon, Iran, Iraq, Kuwait, Lesotho, Saudi Arabia, Oman.

Table 2: Estimation results for 65 countries

	(1)	(2)	(3)	(4)
Sample	No robots sample	Robots sample	No robots sample	Robots sample
Est. method	OLS	OLS	Allerano-Bond	Allerano-Bond
$\log y_{i,t-1}$	0.695***	0.910***	0.419***	0.818***
	(0.045)	(0.014)	(0.077)	(0.017)
$\log s_{i,t}$	-0.056*	0.044***	-0.023	0.081***
	(0.031)	(0.015)	(0.053)	(0.007)
$\log L_{H,i,t}$	-0.053	-0.019	-0.110*	0.043
	(0.125)	(0.070)	(0.056)	(0.029)
$\log L_{L,i,t}$	-0.382	0.087	-0.986	0.519***
	(0.524)	(0.161)	(0.764)	(0.060)
$\log(n+g+\delta)$	0.026	-	0.392***	-
	(0.061)		(0.181)	
$\log(n+\tilde{g}+\delta)$	-	-0.066*	-	-0.010***
		(0.039)		(0.017)
Constant	3.023***	0.744**	-11.240	1.784***
	(0.627)	(0.282)	(14.530)	(0.186)
Country-specific fixed ef.	yes	yes	yes	yes
Time-specific fixed ef.	yes	yes	yes	yes
Observations	366	937	1683	909
R^2	0.881	0.952	-	-
Number of id	38	65	38	65

Note: OLS-estimation: robust standard errors in parentheses; Allerano-Bond estimation: two-step estimation used; ***, **, * denote significance on 1%, 2% and 5% level.

Figure 10 gives estimation results for β_5^{CS} and β_5^{IS} (coefficients corresponding to lagged GDP per capita), value of threshold according to which sample is split is on the horizontal axis. Observe that value of the coefficient is always higher in 'robot sample' than in 'no robots' sample. Until the threshold value of approximately 9 confidence intervals do not overlap. This is true both for OLS and Allerano-Bond estimation. This suggest higher speed of convergence in 'no robots' irrespective of how sample is split into two sub-samples.

Implicit long-run elasticity of income per capita with respect to saving rate is given in Figure 11. Once again, estimation results are consistent with the predictions of the theoretical model. Long run elasticity is higher in the 'robot' sample, the difference in coefficients disappearing around the threshold equal to 3 robots per population of 1 million. However, standard errors (produced using delta-method) from OLS estimates are relatively high and since two confidence intervals overlap, hypothesis of significant differences in the coefficients has to be rejected. On the other hand, Allerano-Bond estimation produces much more precise lower standard errors of the elasticities and confirms that there is a statistically significant difference between them.

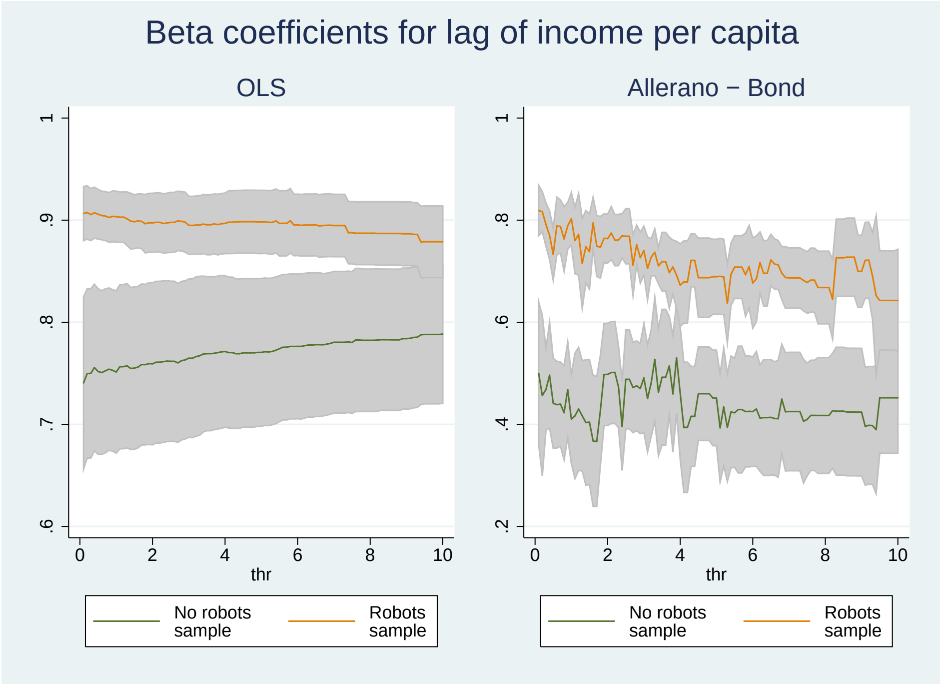

Figure 10: Beta coefficients for lag of income per capita

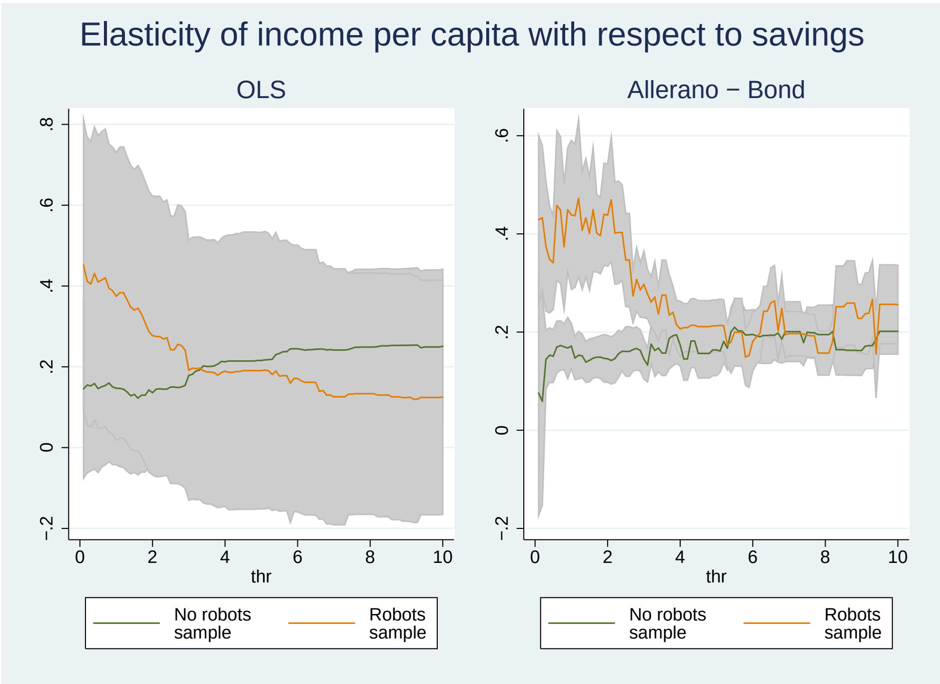

Figure 11: Long-run elasticity of income per capita with respect to saving rate

6 Conclusion

We study the transition to production using automation capital which substitutes low-skilled workers but many tasks can still be performed only by high-skilled labour and are not automatable. Under these assumptions, automation capital does not generate endogenous growth without technological progress. However, technological progress augmenting both effective number of workers and effective number of industrial robots increases rate of long-run growth in an economy with automation capital.

We show that assuming that i) countries differ in fundamentals (for example in total factor productivity) and ii) there is a convergence in fundamentals (for example differences in TFP are decreasing), it is possible that coefficient of σ-convergence evolves in non-linear fashion. In particular, we show that robot-technology can be a cause of temporary divergence in output per capita what is consistent with the data.

We derive a set of estimable equations which allow us to test hypotheses suggested by the model in a Mankiw-Romer-Weil framework. We show that the speed of convergence towards a steady state in an economy with robots is lower than the convergence towards a pseudo steady state where robots are not employed in the production. These differences are robust to different sample splits and estimators. We also find evidence in favour of the hypothesis that elasticity of output with respect to savings rate is higher if robots are used in the production.

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Appendix - Full sample estimations (111 countries)

Table 3: Estimation results for 111 countries

	(1)	(2)	(3)	(4)
Sample	No robots sample	Robots sample	No robots sample	Robots sample
Est. method	OLS	OLS	Allerano-Bond	Allerano-Bond
$\log y_{i,t-1}$	0.901***	0.910***	0.474***	0.818***
	(0.021)	(0.014)	(0.011)	(0.017)
$\log s_{i,t}$	-0.0109	0.044***	-0.044***	0.081***
	(0.016)	(0.015)	(0.007)	(0.007)
$\log L_{H,i,t}$	0.007	-0.019	-0.051***	0.043
	(0.026)	(0.070)	(0.013)	(0.029)
$\log L_{L,i,t}$	-0.174	0.087	0.166*	0.519***
	(0.280)	(0.161)	(0.098)	(0.060)
$\log(n+g+\delta)$	0.110***	-	0.129***	-
	(0.027)		(0.011)	
$\log(n+\tilde{g}+\delta)$	-	-0.066*	-	-0.010***
		(0.039)		(0.017)
Constant	1.095***	0.744**	4.767***	1.784***
	(0.160)	(0.282)	(0.111)	(0.186)
Country-specific fixed ef.	yes	yes	yes	yes
Time-specific fixed ef.	yes	yes	yes	yes
Observations	1793	937	1683	909
R^2	0.927	0.952	-	-
Number of id	111	65	111	65

Note: OLS-estimation: robust standard errors in parentheses; Allerano-Bond estimation: two-step estimation used; ***, **, * denote significance on 1%, 2% and 5% level.

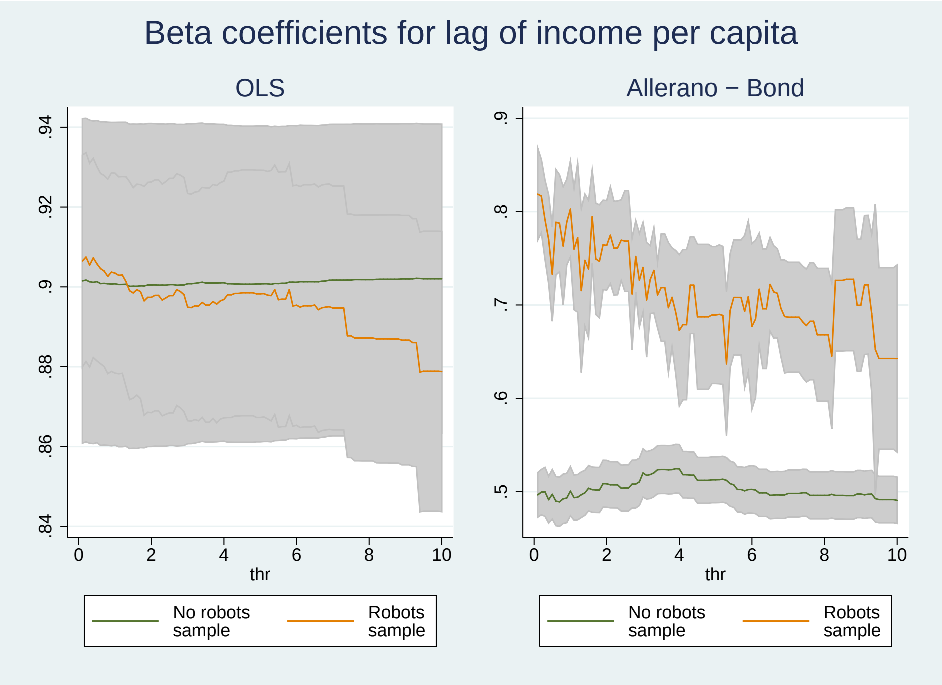

Figure 12: Beta coefficients for lag of income per capita, 111 countries

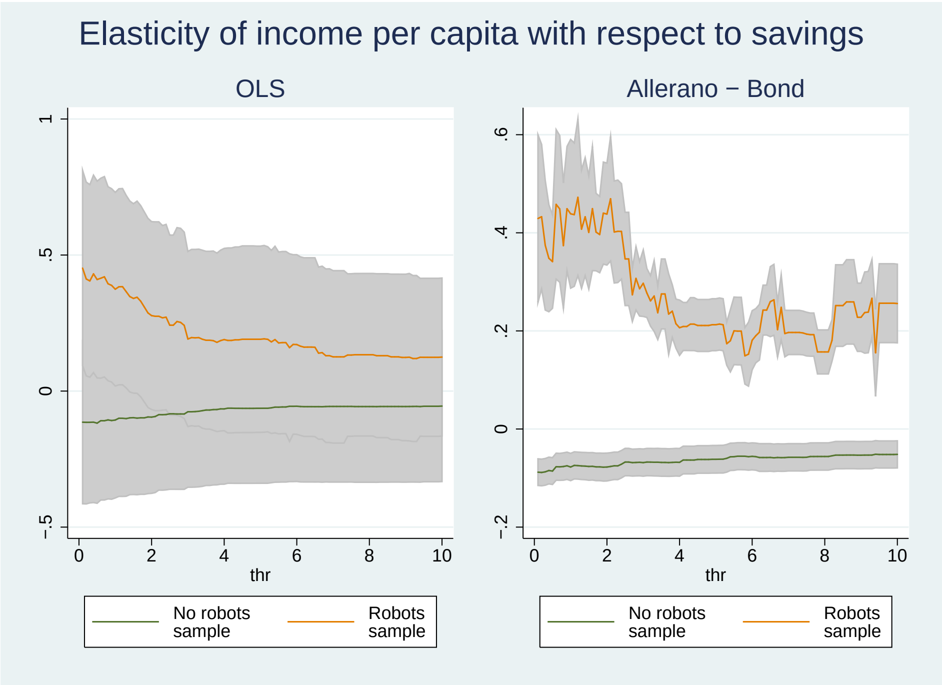

Figure 13: Long-run elasticity of income per capita with respect to saving rate, 111 countries