Department of Economic Policy
Working Paper Series

WP No. 16

Revisiting the Efficiency-Equity Trade-off: A Muli-objective Linear Problem combined with an extended Leontief Input Output - Model

Authors: Luptáčik, M. – Mahlberg, B.

Date: December 19, 2018

Published by:

University of Economics in Bratislava, Department of Economic Policy, Dolnozemská cesta 1, 852 35 Bratislava

Periodicity: irregular

Peer-reviewed

ISSN 1339 - 0430

Revisiting the Efficiency-Equity Trade-off: A Muli-objective Linear Problem combined with an extended Leontief Input Output - Model[1]

Mikuláš Luptáčik[a,b,c] and Bernhard Mahlberg[b,c,#]

[a] University of Economics Bratislava, Bratislava, Slovak Republic

[b] Institute for Industrial Research (IWI), Vienna, Austria

[c] Vienna University of Economics and Business (WU Wien), Vienna, Austria

Abstract

In recent years there has been increasing interest in the question of how inequality affects economic growth. This growing interest has recently stimulated new theoretical as well as empirical research. Some existing theoretical models propose income inequality is detrimental to growth, but alternative theoretical models point at inequality as a determinant furthering economic growth. The main goal of this paper is to obtain deeper insights into the so-called efficiency-equity trade-off. Recently the Stiglitz-Report (Stiglitz et al., 2010) revealed several limits of GDP as an indicator of economic performance and social progress and recommended to shift emphasis towards measuring people's well-being. Following this recommendation, we develop a new multiple criteria decision making model coupled with an extended Leontief input-output model taking into account the social dimension and obtain deeper insights into the so-called efficiency-equity trade-off.

Keywords: Welfare beyond GDP, Utilitarian Social Welfare Function, Rawlsian Social Welfare Function

JEL Classification: C61, C67, D63, I31

---

[1] The authors gratefully acknowledge the valuable comments received on earlier versions of this paper from Martin Lábaj, Eduard Nežinský, and Alexander Schnabl as well as from seminar participants at the University of Economics Bratislava and the participants of the 10th Input-Output Workshop 2018 in Bremen (Germany). The paper is a part of research project APVV-15-0765 „Inequality and economic growth". The usual disclaimer applies.

[#] Corresponding author: bernhard.mahlberg@wu.ac.at, Mahlberg@iwi.ac.at

1. Introduction

The welfare of a nation can scarcely be inferred from a measurement of national income. (S. Kuznets, 1934)

Since the seminal work of Kuznets (1955) asserting that inequality first rises and later falls as an economy develops and its specification as an inverted-U relationship between inequality and the level of per capita income, it has been widely acknowledged that a country's level of economic inequality can be viewed as an outcome of its economic performance. In more recent years there has been increasing interest in the opposite causality (i.e. in the impact of inequality on economic growth), the question of how inequality affects economic growth. This growing interest has recently stimulated new theoretical as well as empirical research. Some existing theoretical models propose inequality is detrimental to growth, but alternative theoretical models point at income inequality as an essential determinant furthering economic growth. Benabou (1996) and Aghion et al. (1999) provide excellent surveys of this theoretical literature. The line of reasoning in these papers focuses on whether countries will face trade-offs between reducing inequality and improving their growth performance, or instead whether there exists a virtuous circle in which growth leads to lower inequality, and lower inequality in turn leads to faster growth. These divergent theoretical deductions have important policy implications, because stimulating economic growth as well as obtaining a reasonably egalitarian income distribution is at the heart of the efficiency-equity trade-off that shapes policy discussions in most countries around the world.

The mechanisms linking inequality and growth have also been addressed in an empirical literature that has developed from the said theoretical models (see Campano and Salvatore, 2006, for an excellent review). Early studies are based on the estimation of cross-country growth regressions in which some measure of inequality is added to the set of explanatory variables. Based on this approach, studies such as Alesina and Rodrik (1994), Persson and Tabellini (1994), Clarke (1995), Alesina and Perotti (1996), Perotti (1996), Deininger and Squire (1998), Barro (2000), Mo (2000), Panizza (2002), Banerjee and Duflo (2003), Foellmi and Zweimüller (2003), Easterly (2007), Sukiassyan (2007), Berg et al. (2012), Rissoet et al. (2013), Cingano (2014), and Policardo et al. (2016) provide a fairly robust body of evidence for a negative relationship between income inequality and economic growth. However, other studies found a positive relationship. The availability of data on income distribution for a larger sample of countries and a longer time span has allowed researchers to explore the issue by means of more sophisticated econometric techniques, and frequently evidence is provided for a positive correlation between income inequality and economic growth (e.g. Li and Zou, 1998; Forbes, 2000).

Arguably, the evidence constitutes a theoretical and empirical puzzle. It is fair to say that no general consensus has emerged so far. Conclusions seem to depend on theoretical preferences and as far as empirical studies go, on the econometric method employed, the countries considered, and the type of income distribution data used (De Dominicis et al., 2008). Another explanation for such discordance is that the negative relationship is found for less developed countries whereas a positive one is found for developed ones (Barro, 2000). Finally, yet another alternative explanation is that growth rates first rise and then decline with an initial inequality (Chen, 2003).

If inequality has a critical effect on economic growth, then it would be cogent to ask what the channels are through which inequality affects growth. A classical analysis of Kaldor (1956) argued that income distribution has a critical effect on capital accumulation, through which economic growth is affected. Mo (2000) investigated plausible channels such as human capital and political stability when the impact of income inequality on growth is considered. In Foellmi and Zweimüller (2003) two major channels are analysed. One is accumulation of human capital and the other the imperfect competition on product markets. Besides capital accumulation, technological progress and its diffusion appears to make a contribution to economic growth (Segerstrom, 1991; Yamamura and Shin, 2005). Accordingly, economic growth is considered to be attributed to several channels such as efficiency improvement, technological progress and capital accumulation (Kumar and Russell, 2002).

Taken the above cited literature into account, the main aim of this paper is to develop a new multiple objective linear programming model coupled with an extended Leontief input-output model useful to obtain deeper insights into the so-called efficiency-equity trade-off.

The remainder of the paper is structured as follows. Section 2 reviews the related literature. Section 3 introduces the relationship between efficiency and equity in welfare economics theory. In section 4 the relationship between efficiency and equity is formulated as a multi-objective optimization problem. The section 5 summarizes the results of this study and suggests possible avenues to future research.

2. Literature review

Is there a trade-off or a complementarity between equity and efficiency? Economists' opinion on this topic is far from uniform. One position holds that a just distribution is a necessary condition for a prosperous economic development. Adam Smith turns out to be a prominent advocate of this view. In the Wealth of Nations (Smith, 1776) he writes "No society can surely be flourishing and happy of which the far greater part of the members is poor and miserable." For this view a high income level for few individuals or privileged ranks cannot be a source of development and growth. Societies' goal should be to raise the standard of living of the whole population including the lower classes, this in turn being the basis of a prosperous economy.

Mainstream economics of the past decades takes a somewhat different position. The opinion is that well functioning markets can guarantee efficiency, but the outcome may not necessarily be just one. In the words of Okun (1975): "The trade-off between equity and efficiency is our biggest socio-economic trade-off, we can't have our cake of market efficiency and share it equally." Throughout the post-war period until recently this position seems to have been accepted by the majority of economists. According to this position, economic progress will trickle down to the poor if the wealthy are unimpeded to pursue their goals. The logic of this equity-efficiency trade-off implies that policy makers have the choice between higher living standards on average but high inequality, or a smaller pie but a larger slice for the poor. In other words, more justice in the distribution of income requires sacrificing output. The price of redistributing income is a lower output, or in a dynamic setting, slower long-run growth.

This opinion is not undisputed in the recent literature. While this new literature does not deny that there is some truth in the above arguments, the efficiency-equity trade-off as a valid generalization has been questioned. This challenge of the recent literature is both based on empirical and theoretical developments.

First, the trade-off has been questioned as a matter of empirical fact. There is very little empirical evidence suggesting that initial inequality in the distribution of income and wealth has a positive impact on subsequent long-run growth rates. To the contrary, studies that regress long-run growth rates on inequality indicators using cross-country data find a negative correlation between these two variables. It seems that, if there is a relationship at all, large inequality is harmful for growth (see, e.g., Bertola et al., 2006).

The second reason that casts doubt on the equity-efficiency trade-off comes from the theoretical literature. More and more economists are arguing that inequality itself may have negative incentive effects. High inequality may lead to lower levels of work effort and, what is of crucial importance in a growth context, to restricted incentives and/or opportunities to undertake productive investments in education and innovative activities. This need not imply that the conventional analysis has become theoretically less compelling or even redundant. However, the previously one-sided focus on the negative side of equality and redistribution has been complemented by a more comprehensive view of the incentive problems facing modern economies (see, e.g., Bertola et al., 2006).

Over the past three decades, theoretical work has come up with a substantial number of channels through which inequality may influence economic growth, either in a positive or in a negative direction (see, e.g., Ehrhart, 2009 and Foellmi and Zweimüller, 2017 for recent and comprehensive overviews). These theoretical contributions have made it clear that the impact of inequality is quite complex and likely to depend on, among other things, the specifics of a country (e.g., the stage of economic development; the extent of market failures; the form of government) or the time horizon considered (e.g., medium run vs. long run). This theoretical ambiguity is mirrored in the empirical literature which — mainly based on broad panels of countries — finds both significantly positive and negative effects, and sometimes even no effects at all.

A closer look at the empirical literature reveals an interesting pattern. On the one hand, estimates based on time-series variation only (e.g., estimations relying on first-differences estimators such as those in e.g. Forbes, 2000) find a strong positive impact of inequality. On the other hand, estimates which also exploit the cross-sectional variation in the data, such as the random-effects estimators in Barro (2000), find a negative relationship (and significantly so in samples that exclude rich countries). Such a negative link is also present in earlier studies based on simple cross-country OLS estimates (e.g., Alesina and Rodrik, 1994 as well as Persson and Tabellini, 1994).

Inequality can influence economic growth via several channels. On the one hand, inequality can promote growth by fostering incentives for innovation and entrepreneurship (Lazear and Rosen, 1981), by rising aggregate savings and investments (Kuznets, 1955; Kaldor, 1956), by promoting the realization of high-return projects (Rosenzweig and Binswanger, 1993), by allowing at least a few individuals to accumulate the minimum to accumulate the minimum needed to start businesses and get a good education (Barro, 2000) or by stimulating R&D (Foellmi and Zweimueller, 2006). On the other hand, inequality may hamper growth by promoting expensive fiscal policies (Perotti, 1993); by inducing an inefficient state bureaucracy (Acemoglu et al. 2011); by hampering human capital formation (Galor and Zeira, 1993; Galor and Moav, 2004); by leading to political instability (Alesina and Perotti, 1996; Rodrik, 1999; Keefer and Knack, 2002; Bénabou, 1996); or by undermining the legal system (Glaeser et al., 2003). Most of the positive effects (e.g., those operating through convex savings functions, market imperfections or innovative incentives) rely on purely economic mechanisms. Arguably, these effects materialize relatively fast, in the short or medium run. Most of the negative effects, however, involve the political process, the change of institutions, the rise of socio-political movements, or they operate through changes in educational attainment of the population. Arguably, these effects take time and materialize primarily in the long run. Examples of recent empirical studies on inequality and economic growth are Foellmi and Zweimüller (2003), Halter et al. (2014), and Berg et al. (2018).

3. Efficiency vs. Equity in Welfare Economics

Welfare economics is concerned with the conditions which determine the total economic welfare of a society. In the broadest sense, the welfare of a society depends upon the satisfaction levels of all its individuals or social groups. But almost every alternative economic state to be judged by welfare economists will have favourable effects on some people (or social groups) and unfavourable effects on others.

In welfarist tradition, this implies interpersonal comparability of utility. But there is no obvious way to determine whether individual A or individual B derives more satisfaction from the consumption of a given bundle of goods. In order to dispense with interpersonal comparability of utility Lange (1942) proposed to define the social welfare not as the sum of the utilities of the individuals (a scalar quantity) but as a vector. The utilities of the individuals are the components of this vector. Let there be h individuals (or social groups) in the community and let u_i be the utility of the *i*-th individual (or social group). Total welfare is then the vector

$$u = (u_1, u_2, \ldots, u_h)$$

Each component of u measures the utility or welfare of the corresponding individual or social group.

The main problem many models of welfare economics have is the formulation of a social welfare function (SWF). The discussion related to the welfare criteria for choosing among efficient allocations of resources, in other words between different vectors u moves between the goals of efficiency and equity.

The first pole – *maximum efficiency* – is represented by *utilitarian* SWF, for which the society's welfare is equal to the sum of utilities of the different individuals or social groups. This concept implies that, according to Lange (1942), the so-called *marginal social significance* of the i th individual, $W_i = \frac{\partial W}{\partial u_i}$, is the same for each individual. In other words, an increase in the welfare W_i of a rich person by one unit has the same social value as an increase of the welfare of a poor person by one unit. This type of SWF can provide very unequal allocations of wealth between the individuals.

The second type of SWF was first posed by the philosopher Rawls (1971). He asserts that members of society would choose to depart from perfect equality only on the condition that the worst-off person under an unequal distribution of utilities would actually be better off than under equality. In other words, for a *Rawlsian* SWF, the welfare of the society depends on the utility of only the poorest or worst-off individual or social group.

The use of this kind of SWF will favour the *maximum equity*, but it can provide poor aggregate performance in terms of overall social welfare. The Rawlsian criterion suggests that many efficient allocations may not be socially desirable and that societies may choose equality even at considerable efficiency cost.[2]

To provide a compromise between efficiency and equity, Romero (2001) proposed a general model in which the views underlying both the utilitarian and Rawlsian criteria are taken into account simultaneously. The following notation is used: $x=(x_1, x_2,..., x_m)$ denotes a vector of policy instruments, $b=(b_1, b_2,..., b_z)$ is a vector of model parameters, and $u=(u_1,u_2,...,u_m)=(h_1(x,b),h_2(x,b),...,h_h(x,b))=h(x,b)$ measures the policy outcome for the corresponding individual or social group. The utility possibility frontier, or the feasible domain of Pareto-efficient policies, is described by $T(u_1,u_2,...,u_h)=k$, and the social welfare function by $W(h(\mathrm{x}, \mathrm{b}))$. The utilitarian SWF and the Rawlsian SWF then take the forms

$$W_U(h(\mathrm{x}, \mathrm{b}))=\sum_{i=1}^{h} h_i(\mathrm{x}, \mathrm{b}) \text{ and } W_R(h(\mathrm{x}, \mathrm{b}))=\underset{i}{Min}[h_i(\mathrm{x}, \mathrm{b})].$$

The model proposed by Romero (2001) is the following multi-objective optimization problem:

$$\begin{gathered}\text{Maximize} \quad W_U(h(\mathrm{x}, \mathrm{b})) \\ \text{Maximize} \quad W_R(h(\mathrm{x}, \mathrm{b})) \\ \text{Subject to } T(u_1,u_2,...,u_h)=k\end{gathered}$$

In this way we are faced with multiple objective decision problems. A wide array of theoretical and empirical contributions to the multiple criteria decision analysis (MCDM) can be found of the literature in the past three decades (for a very good survey see Gal et al. (1999)). In the book by Ballestero and Romero (1998) is referred to a bibliographical survey by Steuer et al. (1996) revealing more than 1200 reviewed journal articles published on multiple criteria decision making just between 1987 and 1992. The majority of applications are at a micro level (project and plan evaluation, capital budgeting, financial and investment planning, marketing policy, etc.) But, also in macroeconomic policy analysis the treatment of multiple goals has already has a long history (see, e.g., Luptacik, 2010, chapter 7).

A possible approach in the evaluation of economic policy using multi-objective optimization models is provided in the book by André et al. (2010). These authors present an integration of economic and environmental criteria for designing efficient policies based on general equilibrium models. Following multi-criteria logic, they construct an operational form of the social welfare function that can be seen as a combination of the

[2] A recent contribution to the issue of efficiency vs. equity for welfare economics is provided by Nicola (2013).

Benthamite or utilitarian social welfare function, which aggregates the (weighted) utilities of all members of society, and of the Rawlsian social welfare function, which considers just the well-being of the worst-off member of society. They show how to formulate a flexible social utility function that takes both issues (total weighted utility and minimum utility) into account.

Based on the afore-presented theory the research questions are:

- How to **operationalize the idea of Romero** (2001) to combine the utilitarian and the Rawlsian Social Welfare Function (SWF)?
- How to model the so-called efficiency – equity trade-off in an empirically meaningful way?
- What are the main **determinants or drivers of the efficiency-equity trade-off**?
- How can economic and social politics influence the efficiency-equity trade-off?

4. Efficiency vs. Equity as a Multi-Objective Optimization Problem

Following the idea of Romero (2001) to combine the utilitarian and the Rawlsian SWF we intend to develop a multi-objective linear programming (MOLP) model coupled with an extended Leontief input-output model.[3] The MOLP-model operationalizes the issue of efficiency and equity by means of explicit multiple criteria as expressing distinct perspectives to evaluate economic policy actions. The augmented Leontief input-output model describes the technology. For supporting policy decision-making, it is particularly relevant to assess the trade-off between efficiency and equity through the use of consistent tools. In this context, the use of multi-objective programming models coupled with input-output analysis seems particularly suited for economic and social policy design and evaluation.

Input-output tables describe a coherent and integrated set of macroeconomic accounts forming a basis for economic analysis and policy formulation. In this context, input-output analysis is an adequate analytical tool for modelling of the inter-relations between different economic activities. The use of the input-output analysis in the framework of linear programming models allows obtaining value-added information, which would not be possible to achieve with the separate use of both techniques. It allows inter alia designing the production possibility frontier (see e.g. Mahlberg and Luptacik, 2014; Luptacik and Mahlberg, 2016). Coupling input output models with MOLP models allows studying different efficient possibilities of production (i.e. output levels for each activity sector for which no other feasible solution exists that allows improving the value of a

[3] Such models have been often applied in the context of economic-social-environmental-energy analysis (see a recent review Oliveira et al., 2016).

given objective function without worsening the value of, at least, one other objective function) consistent with complementary or substitutive goals of economic policy.

We start with the basic Leontief input-output model (Leontief, 1951) where we distinguish n industries in the economy producing n commodities. The industries face the n-dimensional column vector of final demand $\mathbf{f}$ and the induced intermediate demand. The equilibrium output vector is given by

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} \text{ or } (\mathbf{I} - \mathbf{A})\mathbf{x} = \mathbf{f}$$

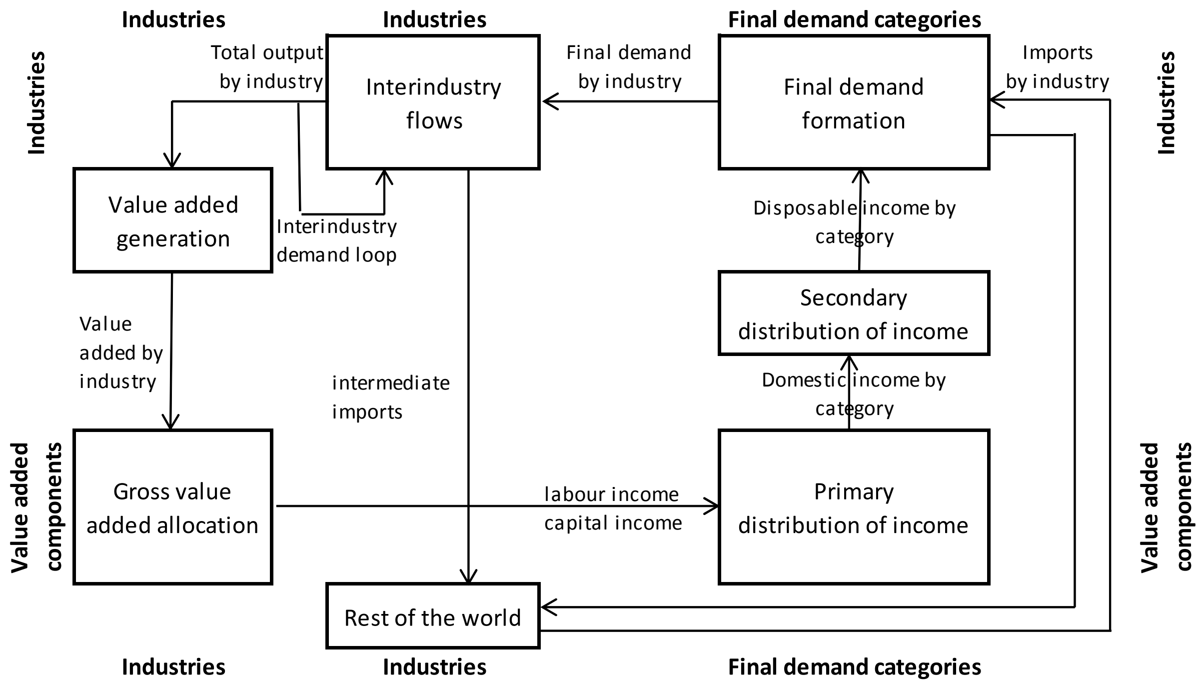
where $\mathbf{A}$ is the $n \times n$-dimensional technical coefficient matrix, $\mathbf{I}$ the $n \times n$-dimensional identity matrix, and $\mathbf{x}$ the n-dimensional column vector of total output.[4] The matrix $(\mathbf{I} - \mathbf{A})^{-1}$ is usually referred to as the Leontief multipliers matrix. Its elements represent the total amount of goods or services directly and indirectly needed to deliver a unit of final demand. This model answers the question how final demand is translated to corresponding output in the industrial sectors of the economy (see e.g. Miller and Blair, 2009).

In the tradition of the Miyazawa approach (Miyazawa, 1976) and similar to Luptacik and Schmoranz (1980, 1989), Payatt (2001), and Ciachini and Socci (2006) the basic Leontief model will be extended by integrating secondary income distribution and by endogenizing household consumption. Using this model the interaction between production and income distribution will be analysed. The Figure 2 shows a diagram where the fundamental mechanism of production and income distribution is shown in terms of interaction between industries, final demand and production factors.

In Figure 2 each arrow identifies an expenditure flow and each box a matrix transformation of one flow into another. The depicted loop consists of logical phases. The production process, that takes place at industry level ("inter-industry flows"), generates total output and value added ("value added generation"). Value added is allocated to production factors (or value added components) labour and capital ("value added allocation"). These value added components constitute total (gross) household income. Total gross household income is then distributed to households of different income classes ("primary distribution of income"). The income by household classes is then redistributed among them through taxation and social transfers to generate disposable income ("secondary distribution of income"). Finally, disposable income generates consumption of private households which are transformed into final demand by industries ("final demand formation").

Figure 2: Extended output-income circular flow

[4] In this section we use lower-case italic letters for scalars, lower-case bold letters for (column) vectors, and upper case bold letters for matrices. The superscript apostrophe means the transposition of a vector. The roof indicates a diagonal matrix.



S.: Ciachini and Socci (2006)

Based on this logical sketch of the extended circular flow, we can define the structural parameters representing the distribution matrices. If we introduce income distribution by private households in the inter-industry model, household consumption is no longer exogenous. The model proposed is built under the assumption of fixed prices and constant distributive shares and coefficients. The distributive shares and the distributive coefficients describe the behaviour of the agents that we consider in the model. These are consumption shares, primary and secondary income distribution shares, shares of value added generated, import and technical coefficients of production. This procedure is a common practice in multi-sectoral applied models for policy and in particular in the case of social accounting matrix models (see e.g. Round, 2003).

We distinguish different final demand categories (i.e., consumption by households, consumption by government, fixed capital formation and exports) and differentiate between endogenous and exogenous final demand. Consumption by households is modelled as endogenous and decomposed into consumption by income classes. Since we differentiate the household consumption by income we denote the consumption by the $n \times h$-dimensional matrix $\mathbf{F}^{pk}$ containing the consumption vectors of the h different types of households where h is the number of income classes considered in our model. Total household consumption is obtained by $\mathbf{F}^{pk}\mathbf{e}_h$ where $\mathbf{e}_h$ is a h-dimensional column vector of ones (identity vector). Consumption by government, fixed capital formation, and exports are considered as exogenous and summarized by $\mathbf{f}^{ex}$ which is a n-dimensional column vector. Based on this notion we rewrite the basic Leontief model as follows

$$(\mathbf{I}-\mathbf{A})\mathbf{x}-\mathbf{F}^{pk}\mathbf{e}_h=\mathbf{f}^{ex}$$

This equation represents the use of goods and services in the economy. The production of commodities requires production factors (e.g. labour, capital, etc) which are not produced by the system and considered as primary inputs. We analyse the impact on employment and distinguish between employed and self-employed. The availability of labour in the economy is restricted by labour supply which is denoted by l. The resource restriction for labour is given by

$$\mathbf{e}_n'\mathbf{l}_1 + \mathbf{e}_n'\mathbf{l}_2 \leq l$$

where $\mathbf{e}_n'$ is the n-dimensional row vector of ones (unity vector), $\mathbf{l}_1$ the n-dimensional column vector of employed and $\mathbf{l}_2$ the n-dimensional column vector of self-employed in the n industries.[5] The generation of employment is denoted by

$$\hat{\mathbf{A}}_{l_1}\mathbf{x} = \mathbf{l}_1,\ \ \hat{\mathbf{A}}_{l_2}\mathbf{x} = \mathbf{l}_2$$

where $\hat{\mathbf{A}}_{l_1}$ and $\hat{\mathbf{A}}_{l_2}$ are the diagonal matrixes of labour input coefficients for employed and self-employed, respectively. The labour input coefficients indicate the labour needed to produce one unit of gross output in each sector. The formulas shown above represent the production sphere of the economy. Now, the question arises how household income is generated, distributed and spent for household consumption.

Income originates from value added, which is determined by subtracting the sum of intermediate purchases by industry from the industry total output. The gross value added generation is given by

$$\hat{\mathbf{A}}_v\mathbf{x} = \mathbf{v}$$

where $\hat{\mathbf{A}}_v$ ($n \times n$-matrix) is a diagonal matrix of value-added coefficients and $\mathbf{v}$ the n-dimensional column vector of value added[6] in n industries. The value-added coefficient of a sector indicates the value added created when one unit of output is produced in the respective sector. Value added is allocated to value-added components (i.e., wages and salaries, gross operating surplus, depreciation of fixed assets, indirect taxes, etc.). Since we endogenize household consumption and consider all other types of final demand as exogenous we focus on labour income and capital income which constitute the two main sources of primary income of private households. The allocation of value added to these two components is given by the following equation

$$\mathbf{m}_1'\mathbf{l}_1 + \mathbf{m}_2'\mathbf{l}_2 + \mathbf{m}_3'\mathbf{v} = b$$

[5] For the sake of simplicity, we confine ourselves on labour as primary input. Of cause, the model can easily be augmented by additional primary factors like e.g. (physical) capital, imports, primary energy, and other natural resources (see, e.g., ten Raa, 2005).

[6] By value added we mean net value added in the sense of the System of National Accounts.

where the n-dimensional row vector $\mathbf{m}_1'$ represent the rates of employed earnings (wages and salaries per unit of employed) in the n industries, the n-dimensional row vector $\mathbf{m}_2'$ the rates of self-employed earnings (income of self employed per unit of self-employed) in the n industries, the n-dimensional row vector $\mathbf{m}_3'$ the shares of capital income on value added of the n industries and b total primary household income. Thus, $\mathbf{m}_1'\mathbf{l}_1$ is earned income of employed (i.e., wages and salaries), $\mathbf{m}_2'\mathbf{l}_2$ earned income of self-employed (i.e., income of tradesmen, farmers and freelance professionals) and $\mathbf{m}_3'\mathbf{v}$ income gained from capital (i.e., interest payments, dividends, rents, etc.).

Primary distribution of income requires the attribution of labour and capital income to the owners of those factors, namely households by income classes. This attribution is given by

$$\hat{\mathbf{B}}_p = \hat{\mathbf{Q}}b$$

where $\hat{\mathbf{B}}_p$ denotes the diagonal matrix of primary households income. The elements of diagonal matrix $\hat{\mathbf{Q}}$ ($h \times h-$ dimensional diagonal matrix, h ... number of private households classes) represent the constant shares of primary income attributed to each household class.

In order to determine the disposable household income by income classes we need to correct household incomes by (income) taxes and (social) transfers (secondary income distribution). This will be determined as follows

$$\hat{\mathbf{Y}} = \hat{\mathbf{B}}_p - \hat{\mathbf{T}}$$

where the $h \times h$-dimensional diagonal matrix $\hat{\mathbf{Y}}$ represents the disposable income of households by income classes, the $h \times h$-dimensional diagonal matrix $\hat{\mathbf{T}}$ the net income taxes (taxes minus monetary social transfers[7]) that each household class receives/forwards from/to the other households classes.

Consumption of private households by industry is given by

$$\mathbf{F}^{pk} = \mathbf{C}(\mathbf{I} - \hat{\mathbf{S}})\hat{\mathbf{Y}}$$

where $\mathbf{F}^{pk}$ ($n \times h$-dimensional matrix) is the private consumption (endogenous part of final demand). In the $n \times h$-dimensional $\mathbf{C}$ matrix each column denotes the commodity structure of private consumption of different income classes (i.e., consumption of the commodity produced by an industry as a share of total consumption). The $h \times h$-dimensional diagonal matrix $\hat{\mathbf{S}}$ represents the saving propensities by households by

[7] Including pensions, family subsidies/child benefits, unemployment benefit, etc. but excluding basic services, such as health care and education.

income classes (taking into household savings).[8] The $\mathbf{I}$ matrix is a $h \times h$-dimensional identity matrix.

Combining the three afore-mentioned equations the following single equation for private consumption can be formulated

$$\mathbf{F}^{pk} = \mathbf{C}(\mathbf{I} - \hat{\mathbf{S}})(\hat{\mathbf{Q}}b - \hat{\mathbf{T}}) .$$

In order to analyse the equity efficiency trade-off in the spirit of Romero (2001) and André et al. (2010) we reformulate the model as multiple-objective linear programming model where social welfare is defined based on consumption of private households by income classes:

$$\text{Maximise } \mathbf{F}^{pk}\mathbf{e}_h \quad \text{(Utilitarian SWF)}$$

$$\text{Maximise } \boldsymbol{\beta} \quad \text{(Rawlsian SWF)}$$

$$\begin{array}{llr}
\text{s.t.} & (\mathbf{I}-\mathbf{A})\mathbf{x} - \mathbf{F}^{pk}\mathbf{e}_h = \mathbf{f}^{ex}, & (1) \\
& \hat{\mathbf{A}}_{l_1}\mathbf{x} - \mathbf{l}_1 = 0, & (2) \\
& \hat{\mathbf{A}}_{l_2}\mathbf{x} - \mathbf{l}_2 = 0, & (3) \\
& \mathbf{e}_n'\mathbf{l}_1 + \mathbf{e}_n'\mathbf{l}_2 \le l, & (4) \\
& \hat{\mathbf{A}}_{v}\mathbf{x} - \mathbf{v} = 0, & (5) \\
& \mathbf{m}_1'\mathbf{l}_1 + \mathbf{m}_2'\mathbf{l}_2 + \mathbf{m}_3'\mathbf{v} - b = 0, & (6) \\
& \mathbf{F}^{pk} - \mathbf{C}(\mathbf{I} - \hat{\mathbf{S}})(\hat{\mathbf{Q}}b - \hat{\mathbf{T}}) = 0, & (7) \\
& \boldsymbol{\beta} - (\mathbf{e}_n'\mathbf{C}(\mathbf{I} - \hat{\mathbf{S}})(\hat{\mathbf{Q}}b - \hat{\mathbf{T}})\hat{\mathbf{H}}^{-1})' \le 0, & (8) \\
& \mathbf{x},\ \mathbf{F}^{pk},\ \boldsymbol{\beta},\ \mathbf{l}_1,\ \mathbf{l}_2,\ \mathbf{v}\ \ b \ge 0, &
\end{array}$$

where $\mathbf{F}^{pk}$ denotes the consumption of different household classes ($n \times h$-dimensional matrix), $\boldsymbol{\beta}$ the per capita consumption of persons living in the worse-off household (n-dimensional vector), $\hat{\mathbf{H}}^{-1}$ the ($h \times h$-dimensional) diagonal matrix of number of persons living in households of different household classes (i.e., number of households of different household classes times average size of households).[9]

The first objective function represents the Benthamite or Utilitarian function where the welfare of the society is equal to the sum of the utilities of the different social groups (i.e., sum of consumption of households by different income classes). The use of this type

[8] Our model is purely static why it does not explain investments in physical capital (i.e., capital allocation) as well as other uses of savings.

[9] A natural extension would be adding the social non-monetary transfers (or social benefits in kind) which is part of government consumption according to the system of national accounts.

allows for the maximization of the overall utility (what we can understand as a *maximum efficiency solution*), but can provide very unequal allocation of wealth (cp. André et al., 2010, p. 128). The second objective function represents Rawlsian function, where the welfare depends only on the utility of the poorest or worst-off social group (i.e., consumption of the worst-off household). The use of this type allows for the optimization of the distributive aspects (what we can interpret as *maximum equity*), but can provide poor aggregate performance in terms of overall social welfare (cp. André et al., 2010, p. 129). We present here a model in which utility as satisfaction received from consuming goods and services. It would be possible to formulate a similar model based on disposable household income which would consider earnings as the source of social welfare (cf. Nicola, 2013).

Constraint (1) reflects the basic Leontief model with final demand disaggregated in an endogenous and an exogenous part. Constraints (2) and (3) model the generation of employment and (4) the resource restriction concerning labour demand and labour supply. The constraints (1) to (4) describe the production sphere whereas the other four constraints represent income generation, income distribution and consumption. Constraints (5) and (6) show the generation of value added and its allocation to owners of the production factors labour and capital. Constraint (7) distributes the income to households differentiated by income classes via primary and secondary income distribution and shows how disposable income is used for consumption. The final constraint (8) models the per capita consumption of households by income classes. The factor $\boldsymbol{\beta}$ is identical for all types of households. Thereby, the constraint of the household with the lowest consumption (i.e., poorest or worst-off household) binds which means that the consumption of the poorest are maximised (cf. min-max principle).

Several methods have been proposed in the scientific literature to find efficient solutions to the MOLP problem. For our model the interactive methods provide nice possibilities and seem to be particularly suitable. These methods do not postulate a priori the full preference structure of the decision maker. The preferences are implicitly revealed in response to a simple question–answer procedure with the decision maker. The feedback between human and model enables the decision maker to explore more deeply the range of possibilities in his feasible region and how the objectives trade off against one another. In this way, the interactive procedure helps the decision maker to understand better the complex structure of the system and to learn more about the analysed problem. For Details see e.g. Oliveira et al. (2016, section 3.3), Luptacik (2010, chapters 7 and 8) and the literature quoted therein as well as works published in the Journal of Multi-Criteria Decision Analysis.

5. Conclusions

In this paper we introduce a multi-objective optimization model coupled with an extended Leontief input output model to analyse the so-called efficiency-equity trade-off in welfare economics in order to operationalize the idea of Romero (2001) to combine the utilitarian and the Rawlsian Social Welfare Function (SWF). It models the so-called efficiency – equity trade-off in an empirically meaningful way to reveal the main determinants or drivers of the efficiency-equity trade-off. This model enables us to show how economic and social politics can influence the efficiency-equity trade-off.

The introduced model opens up new possibilities for future research. The model can be extended e.g. by the environmental dimension (e.g. green house gas emissions). In this way all three pillars of sustainability, namely economic activities (i.e., wealth; the first pillar), social aspects (i.e., income inequality; the second pillar) and environmental quality (i.e., pollution abatement activities and pollution emissions; the third pillar), can be considered simultaneously. Such an extension would be in line with the literature on the Leontief pollution model (cf. inter alia Luptacik and Böhm, 1999, 2010; Mahlberg and Luptacik, 2014) and according to the literature on environmental extended social accounting matrices (see inter alia Cardenete et al., 2012 and Pal, 2016). Such an augmentation would be beyond the scope of the proposed project and left for future research.

References

Acemoglu, D., Ticchi, D., Vindigni, A. (2011). Emergence and Persistence of Inefficient States. *Journal of the European Economic Association*, 9(2), 177–208.

Aghion, P., Caroli, E. and Garcia-Penalosa, C. (1999). Inequality and economic growth: the perspective of the new growth theories. *Journal of Economic Literature*, 37(4), 1615–1660.

Alesina, A. and Perotti, R. (1996). Income distribution, political instability, and investment. *European Economic Review*, 40(6), 1203–1228.

Alesina A. and Rodrik, D. (1994). Distributive politics and economic growth. *Quarterly Journal of Economics*, 109(2), 465–490.

André, F. J., Cardenete, M. A. and Romero, C. (2010). Designing Public Policies. An Approach Based on Multi-Criteria Analysis and Computable General Equilibrium Modeling. Springer-Verlag Berlin Heidelberg.

Ballestero, E. and Romero, C. (1998). Multiple Criteria Decision Making and its Applications to Economic Problems, Kluwer Academic Publishers, Boston.

Banerjee, A. V. and Duflo, E. (2003). Inequality and growth: What can the data say? *Journal of Economic Growth*, 8(3), 267–299.

Barro, R.J. (2000). Inequality and growth in a panel of countries. *Journal of Economic Growth*, 5(1), 5–32.

Benabou, R. (1996). Inequality and growth. NBER Working Paper no. 5658. (https://www.nber.org/papers/w5658)

Berg, A., Ostry, J. D., and Zettelmeyer, J. (2012). What makes growth sustained? *Journal of Development Economics*, 98(2), 149–66.

Berg, A., Ostry J.D., Tsangarides, C.G. and Yakhshilikov, Y. (2018). Redistribution, inequality, and growth: new evidence. Journal of Economic Growth, 23(3), 259–305.

Bertola, G., Foellmi, R., Zweimüller, J. (2006). Income distribution in macroeconomic models. Princeton University Press, New York.

Campano, F. and Salvatore, D. (2006). Income Distribution. Oxford: Oxford University Press.

Cardenete, M.A., Fuentes-Saguar, P.D. and Polo, C. (2012). Energy Intensities and Carbon Dioxide Emissions in a Social Accounting Matrix Model of the Andalusian Economy. *Journal of Industrial Ecology* 16(3), 378–386.

Chen, B.L. (2003). An inverted-U relationship between inequality and long-run growth. *Economics Letters*, 78(2), 205 – 212.

Ciachini, M. and Socci, C. (2006). Income distribution and output change: a macro multiplier approach, in: Salvadori, N. (ed.) *Economic growth and distribution: on the nature and causes of the wealth of nations*, Edward Elgar publisher, Cheltenham (UK) and Northampton (USA), pp. 247-270.

Cingano, F. (2014). Trends in Income Inequality and its Impact on Economic Growth. OECD Social, Employment and Migration Working Papers, No. 163, OECD Publishing. (http://dx.doi.org/10.1787/5jxrjncwxv6j-en)

Clarke, G. R. G. (1995). More evidence on income distribution and growth. *Journal of Development Economics*, 47(2), 403 – 427.

De Dominicis, L., Florax, R. J. G. M. and De Groot, H. L. F. (2008). A Meta-Analysis on the Relationship between Income Inequality and Economic Growth. *Scottish Journal of Political Economy*, 55(5), 654–682.

Deininger, K. and Squire, L. (1998). New ways of looking at old issues: inequality and growth. *Journal of Development Economics*, 57(2), 259–287.

Easterly, W. (2007). Inequality does cause underdevelopment: Insights from a new instrument. *Journal of Development Economics*, 84(2), 755–76.

Ehrhart, C. (2009). The effects of inequality on growth: a survey of the theoretical and empirical literature, Working Paper 107, Society for the Study of Economic Inequality. (http://www.ecineq.org/milano/WP/ECINEQ2009-107.pdf)

Foellmi, R. and Zweimüller, J. (2003). Inequality and Economic Growth: European versus U.S. Experience, Working Paper No. 158, Institute for Empirical Research in Economics, University of Zurich. (http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.195.8639&rep=rep1&type=pdf)

Foellmi, R. and Zweimueller, J. (2006). Income Distribution and Demand-Induced Innovations. *Review of Economic Studies*, 73(4), 941 - 960.

Foellmi, R. and Zweimueller, J. (2017). Is inequality harmful for innovation and growth? Price versus market size effects. *Journal of Evolutionary Economics*, 27(2), 359-378.

Forbes, K. (2000). A reassessment of the relationship between inequality and growth. *American Economic Review*, 90(4), 869 – 886.

Halter, D., Oechslin, M., and Zweimüller, J. (2014). Inequality and Growth: The Neglected Time Dimension, *Journal of Economic Growth*, 19(1), 81-104.

Gal, T., Stewart, Yh. J. and Hanne, Th. (eds.) (1999). Multicriteria Decision Making. Advances in MCDM Models, Algorithms, Theory and Applications, Kluwer Academic Publishers, Boston.

Galor, O. and Moav, O. (2004). From Physical to Human Capital Accumulation: Inequality and the Process of Development. *Review of Economic Studies*, 71(4), 1001 - 1026.

Galor, O., Zeira, J. (1993). Income distribution and macroeconomics. *Review of Economic Studies*, 60, 35–52.

Glaeser, E., Scheinkman, J., Shleifer, A. (2003). The injustice of inequality. *Journal of Monetary Economics*, 50(1), 199 - 222.

Kaldor, N. (1956). Alternative theories of distribution. *Review of Economic Studies*, 23(2), 83–100.

Kumar, S. and Russell, R. (2002). Technological Change, Technological Catch-up, and Capital Deepening: Relative Contributions to Growth and Convergence. *American Economic Review*, 92(3), 527-548.

Kuznets, S. (1934). National income, 1929-1932. National Bureau of Economic Research, Inc.

Kuznets, S. (1955). Economic growth and income inequality. *American Economic Review*, 45(1), 1–28.

Lange, O. (1942). The Foundations of Welfare Economics. *Econometrica*, 10(3-4), 215 - 228.

Lazear, E. P. and Rosen, S. (1981). Rank-order tournaments as optimum labor contracts. *Journal of Political Economy*, 89(5), 841–64.

Leontief, W. (1951). Input-Output Economics. *Scientific American*, 185(4), 15 - 21.

Li, H. and Zou, H. (1998). Income inequality is not harmful for growth: theory and evidence. *Review of Development Economics*, 2(3), 318 – 334.

Luptacik, M. (2010). Mathematical Optimization and Economic Analysis, Springer Publisher, New York Dordrecht Heidelberg London.

Luptacik, M. and Böhm, B. (1999). A Consistent Formulation of the Leontief Pollution Model. *Economic Systems Research*, 11(3), 263-276.

Luptacik, M. and Böhm, B. (2010). Efficiency analysis of a multisectoral economic system. *Central European Journal of Operations Research*, 18(4), 609-619.

Luptačik, M. and Mahlberg, B. (2016). Productivity change over time in a multisectoral economic system. *Economic Systems Research*, 28(3), 344-361

Luptacik, M. and Schmoranz, I. (1980). Demographic changes and economic consequences: demo-economic multiplier for Austria. *Empirical Economics*, 5(1), 55-67.

Luptacik, M. and Schmoranz, I. (1989). An Extension of a Static Input-Output Model for Demographic-Economic Analysis. In: Wenig A., Zimmermann K.F. (eds) *Demographic Change and Economic Development*. Studies in Contemporary Economics. Springer, Berlin, Heidelberg, pp. 253-271.

Keefer, P. and Knack, S. (2002). Polarization, politics and property rights: Links between inequality and growth. *Public Choice*, 111(1–2), 127–54.

Mahlberg, B. and Luptacik, M. (2014). Eco-Efficiency and Eco-Productivity change over time in a multisectoral economic system. *European Journal of Operational Research*, 234(3), 885-897.

Miller, R.E. and Blair, P.D. (2009). Input-Output Analysis: Foundations and Extensions, 2nd Edition, Cambridge University Press, Cambridge et al.

Miyazawa, K. (1976). Input-output analysis and the structure of income distribution. Springer publisher, Berlin.

Mo, P.H. (2000). Income inequality and economic growth, *Kyklos*, 53(3), 293–315.

Nicola, P.C. (2013). Efficiency and Equity in Welfare Economics, Lecture Notes in Economics and Mathematical Systems 661, Springer-Verlag Berlin Heidelberg.

Okun, A. (1975). Equality and Efficiency. The Big Trade-off, The Brookings Institution, Washington DC.

Oliveira, C., Coelho, D. and Antunes, C. H. (2016). Coupling input–output analysis with multiobjective linear programming models for the study of economy–energy–environment–social (E3S) trade-offs: a review. *Annals of Operations Research*, 247(2), 471–502.

Pal, B.D. (2016). The Conflict Between Economy and Environment. An Environmentally Extended Social Accounting Matrix for India. In: Nautiyal S., Schaldach R., Raju K., Kaechele H., Pritchard B., Rao K. (eds) *Climate Change Challenge (3C) and Social-Economic-Ecological Interface-Building*. Springer, Cham

Panizza, U. (2002). Income inequality and economic growth: Evidence from American data. *Journal of Economic Growth*, 7(1), 25–41.

Perotti, R. (1993). Political Equilibrium, Income Distribution, and Growth. *Review of Economic Studies*, 60(4), 755-776.

Perotti, R. (1996). Growth, income distribution, and democracy: what the data say. *Journal of Economic Growth*, 1(2), 149–187.

Persson, T. and Tabellini, G. (1994). Is inequality harmful for growth? *American Economic Review*, 84(3), 600–621.

Policardo, L., Punzo, L.F. and Sánchez Carrera, E.J. (2016). Unsustainable Inequality: is there a turning point? Working Paper No. 728, Department of Economics and Statistics, University of Siena. (http://repec.deps.unisi.it/quaderni/728.pdf)

Rawls, J. (1971), A Theory of Justice, Harvard University Press, Cambridge, Mass.

Rodrik, D. (1999). Where did all the growth go? External shocks, social conflict, and growth collapses. *Journal of Economic Growth*, 4(4), 385–412.

Rosenzweig, M., Binswanger, H. (1993). Wealth, Weather Risk and the Composition and Profitability of Agricultural Investments. *Economic Journal*, 103(1), 56-78.

Risso, W.A., Punzo, L.F. and Sánchez Carrera, E.J. (2013). Economic growth and income distribution in Mexico: A cointegration exercise. *Economic Modelling*, 35, 708–714.

Romero, C. (2001). A Note on Distributive Equity and Social Efficiency. *Journal of Agricultural Economics*, 52(2), 110-112.

Round, J. (2003). Social Accounting Matrices and SAM-based Multiplier Analysis, in: F. Bourguinon and L.A.P. da Silva (eds.) The impact of economic policies on poverty and income distribution: evaluation techniques and tools, World Bank and Oxford University Press, Washington, DC

Segerstrom, P.S. (1991). Innovation, imitation, and economic growth. *Journal of Political Economy*, 99(4), 807 – 827.

Smith, A. (1776). An Inquiry into the Nature and Causes of the Wealth of Nations, London: W. Strahan.

Steuer, R. E., Gardiner, L. R. and Gray, J. (1996). A Bibliographic Survey of the Activities and International Nature of Multiple Criteria Decision Making. *Journal of Multicriteria Decision Analysis*, 5(3), 195-217.

Stiglitz, J.E., Sen, A. and Fitoussi, J.-P. (2010). Mismeasuring our lives: why GDP doesn't add up. New Press, New York.

Sukiassyan, G. (2007). Inequality and growth: what does the transition economy data say? *Journal of Comparative Economics*, 35(1), 35–56.

ten Raa, T. (2005). The economics of input-output analysis. Cambridge University Press, Cambridge.

Yamamura, E. and Shin, I. (2009). Effects of Income Inequality on Growth through Efficiency Improvement and Capital Accumulation. *International Economic Journal*, 23(2), 237–258.