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Eco-Efficiency and Eco-Productivity change over time in a multisectoral economic system[1]

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Abstract
We measure eco-efficiency of an economy by means of an augmented Leontief input-output model extended by constraints for primary inputs. Using a multi-objective optimization model the eco-efficiency frontier of the economy is generated. The results of these multi-objective optimization problems define eco-efficient virtual decision making units (DMUs). The eco-efficiency is obtained as a solution of a data envelopment analysis (DEA) model with virtual DMUs defining the potential and a DMU describing the actual performance of the economy. In this paper the procedure is extended to an intertemporal approach in the spirit of the Luenberger productivity indicator. This indicator permits decomposing eco-productivity change into eco-efficiency change and eco-technical change. The indicator is then further decompounded in a way that enables us to examine the contributions of individual production factors, undesirable as well as desirable outputs to eco-productivity change over time. For illustration purposes the proposed model is applied to investigate eco-productivity growth of the Austrian economy.

Keywords: Data Envelopment Analysis, Luenberger Indicator, Multi-Objective Optimization, Neoclassical Growth Accounting
JEL codes: C67, O47, Q53, Q57

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1 Introduction

One of the goals of the European Union's strategy for a smart, sustainable and inclusive growth (the so called Europe 2020) is the reduction of CO_2 emissions by 20% compared to 1990 levels (European Commission, 2010). Since a general aim of the economic policy in Europe remains to keep economic growth, a reduction of air pollution requires an increase of eco-efficiency. In this context, increasing eco-efficiency means decoupling pollution (e.g. CO_2 emission) from economic development. Without such a de-linking the environmental target cannot be fulfilled. Another goal of Europe 2020 is the increase of energy efficiency which is defined as a reduction of energy consumption. This reduction clearly implicates also a raise in eco-efficiency. Strengthening eco-efficiency has also been identified by the United Nations Industry and Development Organization (UNIDO) as one of the major strategic elements in its work on sustainability. It constituted a Cleaner and Sustainable Production Unit (UNIDO, 2012a) and started an Eco-efficiency (Cleaner-Production) Program (UNIDO, 2012b).

The concept of eco-efficiency was first described by Schaltegger and Sturm (1989). They defined eco-efficiency as ratio between environmental impact added and value added. Eco-efficiency aims at achieving more goods and service outputs with less resource inputs as well as less waste and emissions. Eco-efficiency is related to sustainability in the sense that the later is a broader notion whereas the former is a new indicator of economic performance. It differs from sustainability in that it takes into account environmental and economic dimensions but does not include social aspects. Eco-efficiency is a necessary but not a sufficient condition for achieving sustainability. Measurement of eco-efficiency is important to determine success (economic and environmental), identify and track trends, prioritize actions and ascertain areas for improvement. Monitoring eco-efficiency on the macro-level is useful in order to make sustainability accountable.

Like in Korhonen and Luptacik (2004), in this paper it is assumed that decision making units (say, countries) want to produce desirable outputs as much as possible and produce minimal undesirable outputs (e.g. pollutions) with less inputs. In contrast, usual analysis of (technical) efficiency defines efficiency as a ratio of a weighted sum of desirable outputs to a weighted sum of inputs, and does not take undesirable outputs into consideration. The concept of eco-efficiency has the advantage over traditional (technical) efficiency that it considers inputs, desirable outputs and undesirable outputs in one model and takes economical as well as ecological aspects simultaneously into account.

The efficiency analysis of any decision making unit (DMU) without taking economic as well as ecological issues into account often yields erroneous inferences concerning the real health of the DMU. This is precisely because there always exists a trade-off between economy and environment, and an economy's performance is not sustainable without a healthy ecological system. Because win-win solutions for economy and ecology seem quite elusive in practice, there arises the concept of trade-offs and efficiency frontiers for economy. Therefore, there is a need to have a measure of performance characterized by an eco-efficiency frontier that aims at providing efficient solutions in relation to the objective of optimizing the goals of economy as well as ecology. That is, DMUs lying on the eco-efficiency frontier cannot increase the output of economic goods and services without increasing at least one input or increasing waste and emissions. These DMUs are efficient in sense of Koopmanns (1951). As is known from the literature (see e.g. Färe et al., 1989, 1996; Tyteca, 1996, 1997; and Sahoo, et al., 2011), the nonparametric methodology of data envelopment analysis (DEA) helps estimating the eco-efficiency frontier. Particularly in the context of eco-efficiency analysis, the main challenge is the lack of measures like market prices for undesirable outputs to be used as weights to aggregate various inputs, desirable outputs and undesirable outputs. Although various techniques for eco-efficiency measurement have been presented in the literature, most eco-efficiency measures are either very limited or depend on subjective arbitrary weighting scheme. The technique of DEA endogenously generates the most favourable weights that maximize the relative efficiency of the evaluated DMU in comparison with the maximum attainable efficiency. This means that DEA presents every evaluated DMU in its most favourable environment.

In the paper by Luptacik and Böhm (2010) eco-efficiency of a whole economy is measured by means of an augmented Leontief input-output model extended by constraints for primary inputs. Using multi-objective optimization models an eco-efficiency frontier of the economy is generated. The solutions of the multi-objective optimization problems define eco-efficient virtual decision making units (DMUs). The eco-efficiency of the economy can be obtained as a solution of a DEA model with the virtual DMUs defining the potential and a DMU describing the actual performance of the economy. This model allows us taking into account the interdependences of the sectors in an economy in eco-efficiency analyses. Furthermore, it permits estimating eco-efficiency of an economy with respect to its own potential and without the need to compare it with other economies – economies that may possess different technologies and varying mutual interdependencies due to international trade.

This model, however, is purely static and cannot account for eco-efficiency change (catch-up) or explain changes in eco-technical (frontier shift) over time. One main aim of this study is to extend the static eco-efficiency analysis to an intertemporal setting. For this purpose the Luenberger productivity indicator is utilized, which was introduced by Chambers et al. (1996a, 1996b). This indicator measures productivity change (*PRODCH*) and permits decomposing it into change in efficiency (*EFFCH*) on the one hand and change in the frontier technology, i.e., technical change (*TECHCH*) on the other.

This measure differs from the more frequently applied Malmquist productivity index in two primary ways. Firstly, it is constructed based on directional distance functions, which simultaneously adjust outputs and inputs in a direction chosen by the investigator, and, secondly, it has an additive structure, i.e. it is expressed as differences rather than ratios of distance functions. Contrary to several other indexes and indicators applied in productivity studies (e.g. Fisher index, Törnqvist index, Bennet–Bowley indicator) the proposed measure does not demand price information at any stage.

The Luenberger indicator itself is not capable of attributing eco-productivity change to changes in use of production factors or in production of undesirable or desirable outputs. To overcome this limitation our indicator is decomposed in a way that enables one to examine the contributions of individual production factors and individual (desirable and undesirable) outputs to eco-productivity change. The results allow the inference of which inputs and/or desirable/undesirable outputs of an economy are the drivers of eco-productivity change.

Our paper is structured as follows. Section 2 presents in detail the (static) model of Luptacik and Böhm (2010) and extends this model in line with the directional distance function approach. Section 3 introduces our method to measure eco-efficiency and eco-productivity change over time; whilst Section 4 deals with an illustrative empirical application of the proposed model, with Section 5 left for our concluding remarks.

2 Methodology

2.1 The Leontief input-output model and the augmented Leontief input-output model

The conventional Leontief's input–output model conveniently describes the production relations of an economy in period *t* for a given nonnegative vector of final demands for *n* goods produced in *n* interrelated sectors; gross output of the sectors in period *t* is denoted by the *n*-dimensional vector. Production technology in period *t* is given by a (*n* × *n*) input coefficient matrix. This in turn informs the use of a particular good *i* required for the production of a unit of good *j*. Luptacik and Böhm (2010) introduced a restriction of the use of primary input factors by the available primary input quantities in period *t* in this model.

The conventional Leontief's input–output model has been extended to a model version including pollution generation and abatement activities. The well known augmented Leontief model (Leontief, 1970; see also Lowe, 1979; Luptacik and Bohm, 1999; Miller and Blair, 2009) is written as

$$\begin{bmatrix} I - A_{11,t} & -A_{12,t} \\ -A_{21,t} & I - A_{22,t} \end{bmatrix} \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} \geq \begin{bmatrix} y_{1,t} \\ -y_{2,t} \end{bmatrix} \tag{1}$$

where the following notation is used: $x_{1,t}$ is the *n*-dimensional vector of gross industrial outputs in period *t*; $x_{2,t}$ is the *o*-dimensional vector of anti-pollution activity levels in period *t*; $A_{11,t}$ is the (*n* × *n*) matrix of conventional input coefficients, showing the input of good *i* per unit of the output of good *j* (produced by sector *j*) in period *t*; $A_{12,t}$ is the (*n* × *o*) matrix with $a_{ik,t}$ representing the input of good *i* per unit of the eliminated pollutant *k* (eliminated by anti-pollution activity *k*) in period *t*; $A_{21,t}$ is the (*o* × *n*) matrix showing the output of pollutant *k* per unit of good *i* (produced by sector *i*) in period *t*; $A_{22,t}$ is the (*o* × *o*) matrix showing the output of pollutant *k* per unit of eliminated pollutant *l* (eliminated by anti-pollution activity *l*) in period *t*; *I* is the identity matrix; $y_{1,t}$ is the *n*-dimensional vector of final demands for economic commodities in period *t* (also referred to as net output or desirable output); $y_{2,t}$ is the *o*-dimensional vector of the net generation of pollutants in period *t* which remain untreated after abatement activity (also referred to as tolerated level of net pollution or undesirable output). The *k*-th element of this vector represents the pollution standard of pollutant *k* and indicates the tolerated level of net pollution.

The restriction of the use of primary input factors by the available primary input quantities in the augmented model can be written as

$$\begin{bmatrix} B_{1,t} & B_{2,t} \end{bmatrix} \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} \le z_t \tag{2}$$

In addition, the relation for primary inputs contains $B_{1,t}$ the ($m \times n$) matrix of primary factor requirements per unit of output in period t for production activities in period t, $B_{2,t}$ the ($m \times o$) matrix of primary input coefficients for abatement activities in period t, and available primary input quantities z_t in period t.

In order to model multiple-output multiple-input technologies, the notion of input and output distance functions can be used. For a single output this corresponds to the concept of a production function. Distance functions are well suited to define input and output oriented measures of eco-efficiency. To work out such eco-efficiency measures and to derive the output potential of an economy with n outputs we face in principle a multi-objective optimization problem. In many cases such problems are reduced to a single objective optimization problem by suitable aggregation. For example, ten Raa (1995, 2005) uses world market prices for the n commodities employed in his model to reduce the optimization of n outputs to that of a single sum of values of the net products.

Pursuing the multiple objective approach and in analogy to Luptacik and Böhm (2010) we formulate the multi-objective optimization problem where each net output $y_{1,t}$ is maximized subject to restraints on the availability of primary inputs z_t^0 as follows

$$\begin{aligned} &\operatorname*{Max}_{x} y_{1,t} \\ &\text{s.t} \\ &(I - A_{11,t})x_{1,t} - A_{12,t}x_{2,t} - y_{1,t} \ge 0 \\ &-A_{21,t}x_{1,t} + (I - A_{22,t})x_{2,t} + y_{2,t} \ge 0 \\ &B_{1,t}x_{1,t} + B_{2,t}x_{2,t} \le z_t^0 \\ &x_{1,t}, x_{2,t}, y_{1,t}, y_{2,t} \ge 0 \end{aligned} \tag{3}$$

We use the notation "Max" for a vector optimization problem and "max" for a single objective problem. Luptacik and Böhm (2010) thus solve n single objective problems maximizing final demand for each commodity separately:

$$\max y_{1,t}^j \; (j = 1, \ldots, n) \tag{4}$$

subject to the constraints in (3).

For each of the n solutions of (4) denote the (also n-dimensional) solution vector x_t^{*j} (j = 1,...,n) representing the gross productions of commodities. The respective net output column vectors are denoted $y_{1,t}^{*j}$. Minimization of net pollution under the constraints (3) yields the trivial solution where all variables are zero.

Alternatively for given final demand $y_{1,t}^{0}$ and environmental standard $y_{2,t}^{0}$ (the tolerated level of net pollution) the inputs z_t are minimized.

$$
\begin{aligned}
&\underset{x}{\mathrm{M\,in}}\, z_t \\
&\text{s.t} \\
&(I - A_{11,t})x_{1,t} - \quad A_{12,t}x_{2,t} \qquad \geq y_{1,t}^{0} \\
&\quad - A_{21,t}x_{1,t} + (I - A_{22,t})x_{2,t} \qquad \geq -y_{2,t}^{0} \\
&\qquad B_{1,t}x_{1,t} + \quad B_{2,t}x_{2,t} - z_t \leq 0 \\
&\qquad\qquad x_{1,t}, x_{2,t}, z_t \geq 0
\end{aligned}
\tag{5}
$$

In this case, therefore, *m* single objective problems are solved

$$\min z_t^i \; (i = 1, \ldots, m) \tag{6}$$

subject to the constraints in (5).

The *m* solution vectors x_t^{*i} (*i*=1,...,*m*) describe the optimal gross production values of commodities for given final demand $y_{1,t}^{0}$ and environmental standards $y_{2,t}^{0}$ under the individual minimization of the primary factors *i*=1,...,*m*. The optimal primary input vectors are denoted by z_t^{*i}.

Like Luptacik and Böhm (2010) these sets of values of both problems defined above are arranged column-wise in a pay-off matrix with the optimal values appearing in the main diagonal while the off-diagonal elements provide the levels of other sector net outputs and inputs compatible with the individually optimized ones. This payoff matrix of dimension $(n + o + m) \times (n + m)$ for the augmented model is written in the following way

$$
\begin{bmatrix} Q_{t,t} \\ Z_{t,t} \end{bmatrix} =
\left[\begin{array}{ccc|ccc}
y_{1,t}^{*1} & \cdots & y_{1,t}^{*n} & y_{1,t}^{0} + s_{y_1}^{1} & \ldots & y_{1,t}^{0} + s_{y_1}^{m} \\
y_{2,t}^{1} & \cdots & y_{2,t}^{n} & y_{2,t}^{0} - s_{y_2}^{1} & \cdots & y_{2,t}^{0} - s_{y_2}^{m} \\
z_t^{0} - s_z^{1} & \cdots & z_t^{0} - s_z^{n} & z_t^{*1} & \cdots & z_t^{*m}
\end{array}\right]
\equiv
\begin{bmatrix} Q_{1;t,t} \\ Q_{2;t,t} \\ Z_{t,t} \end{bmatrix}
$$

where the s_j^i ($j = y_1, y_2, z$) represent the respective vectors of slack variables in the optimization of variable *i* ($i = 1,\ldots,n$, $n + 1,\ldots, n + m$). Thus, each column of the payoff matrix containing either the maximal net output of a particular commodity for given tolerated levels of net pollution and for given inputs (the first *n* columns), or the minimal primary input for given tolerated values of net pollution and for given final demand (the last *m* columns) yields an efficient solution (in the sense of Pareto-Koopmans). In this way the eco-efficiency frontier of the economic system can be generated. In other words, the matrix $Q_{t,t}$ includes the combinations of output quantities that are possible to produce for given tolerated levels of net pollution and for any given combination of inputs. In this way the "macroeconomic production function" for multiple-input multiple output technologies can be described.

Each of the points in the payoff-matrix $Q_{t,t}$ is constructed independently of the other points, but takes account of the entire systems relations. Knowing the eco-efficiency frontier the eco-efficiency of the actual economy can be estimated. Each of the columns of the pay-off matrix can be seen as a virtual decision making unit with different input and output characteristics which is using the same production technique. The real economy as given by actual output and input data defines a new decision making unit whose distance to the frontier can be estimated.

For this purpose Luptacik and Böhm (2010) formulate the following input-oriented DEA model, measuring the eco-efficiency of the economy described by the actual output and input data $\left(y_{1,t}^0, y_{2,t}^0, z_t^0\right)$

$$\begin{aligned}
&\min_{\mu} \theta \\
&s.t. \\
&\quad Q_{1;t,t}\mu \geq y_{1,t}^0 \\
&\quad -Q_{2;t,t}\mu \geq -y_{2,t}^0 \\
&\theta z_t^0 - Z_{t,t}\mu \geq 0 \\
&\quad \theta, \mu \geq 0
\end{aligned} \tag{7}$$

where $Q_{1;t,t}$ is the output matrix, $Q_{2;t,t}$ the pollution matrix, $Z_{t,t}$ the input matrix, θ eco-efficiency and μ the intensity vector. The columns of the matrix $Q_{t,t}$ are the virtual DMUs, which represent the points of the eco-efficiency frontier. DMU_0 described by the actual output and input data $\left(y_{1,t}^0, y_{2,t}^0, z_t^0\right)$ is not included in the description of the production possibility set, this is because the eco-efficient points (the virtual DMUs) that enter into the evaluation are unaffected by such a removal. This is also true for an eco-efficient DMU_0 that is on a part of the eco-efficiency frontier, but not an extreme point.

2.2 The relationship between the DEA model and the LP-Leontief model

Like in Luptacik and Böhm (2010) the augmented Leontief model (2) can alternatively be formulated as an LP-problem by minimizing primary inputs for given levels of final demand $y_{1,t}^0$ and environmental standards $y_{2,t}^0$ we get

$$\begin{aligned}
&\min_{x} \gamma \\
&s.t. \\
&\qquad \left(I - A_{11,t}\right)x_{1,t} - A_{12,t}x_{2,t} \geq y_{1,t}^0 \\
&\qquad -A_{21,t}x_{1,t} + \left(I - A_{22,t}\right)x_{2,t} \geq -y_{2,t}^0 \\
&\gamma z_t^0 - B_{1,t}x_{1,t} - B_{2,t}x_{2,t} \geq 0
\end{aligned} \tag{8}$$

$$x_{1,t}, x_{2,t}, \gamma \geq 0$$

The parameter γ provides a radial eco-efficiency measure. It records the degree by which primary inputs could be proportionally reduced but still capable of producing the required net outputs to satisfy the exogenously given final demand.

We rephrase the input-oriented eco-efficiency models (7) and (8) taken from Luptacik and Böhm (2010) to models of non-oriented proportional directional distance function of eco-efficiency in the following way:

$$\begin{aligned} \rho_t\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right) = \max_{\mu,\beta} \; & \beta \\ \text{s.t.} \quad & \\ -\beta y_{1,t}^0 + Q_{1;t,t}\mu & \geq y_{1,t}^0 \\ Q_{2;t,t}\mu & \leq y_{2,t}^0 \\ \beta z_t^0 + Z_{t,t}\mu & \leq z_t^0 \\ \mu \geq 0, \; \beta & \text{ free} \end{aligned} \tag{9}$$

and

$$\begin{aligned} \omega_t\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right) = \max_{x,\delta} \; & \delta \\ \text{s.t.} \quad & \\ -\delta y_{1,t}^0 + \left(I - A_{11,t}\right)x_{1,t} - A_{12,t}x_{2,t} & \geq y_{1,t}^0 \\ A_{21,t}x_{1,t} - \left(I - A_{22,t}\right)x_{2,t} & \leq y_{2,t}^0 \\ \delta z_t^0 + B_{1,t}x_{1,t} + B_{2,t}x_{2,t} & \leq z_t^0 \\ x_{1,t}, x_{2,t} \geq 0, \; \delta & \text{ free} \end{aligned} \tag{10}$$

The models (9) and (10) are based on the directional distance function which was proposed by Chambers et al. (1996b). In models (9) and (10) we consider a special case where we postulate constant returns to scale (CRS) and the direction vector for net outputs equal to the exogenously given level of final demand as well as for primary inputs equal to the exogenously given level of available inputs, which yields us a non-oriented proportional measure of eco-inefficiency. This is a radial measure which considers the proportional reductions in primary inputs and proportional extension of desirable output simultaneously.[2] The objective functions of models (9) and (10), i.e., ρ_t and ϖ_t represent the eco-inefficiency scores of an economy. For an eco-efficient economy $\rho_t = 0$ and $\varpi_t = 0$ and for an eco-inefficient economy $\rho_t > 0$ and $\varpi_t > 0$.

Taking into account the interpretation of the eco-inefficiency parameters β in the DEA model (9) and δ in the Leontief model (10) it can be seen that despite the different model formulations the objective functions are similar. Both models measure the eco-efficiency of the economy by radial

[2] Model (9) and model (10) can be formulated as input-oriented by equating the direction vector of net outputs with 0 or as output-oriented model by equating the direction vectors of primary inputs with 0. For oriented models all statements of the rest of this section pertain analogously.

reduction of primary inputs as well as radial expansion of net output for given amounts of resources and final demand in the economy. Denoting

$$\begin{bmatrix} A_{11,t} & A_{12,t} \\ A_{21,t} & A_{22,t} \end{bmatrix} = A_t, \quad \begin{bmatrix} y_{1,t} \\ -y_{2,t} \end{bmatrix} = y_t \text{ and } \begin{bmatrix} B_{1,t} & B_{2,t} \end{bmatrix} = B_t$$

The relationships between (8) and (9) are given by the following proposition.

Proposition 1: *Assuming the workability of the Leontief system and the indecomposability*[3] *of the input coefficient matrix* $A_{11,t}$ *are fulfilled the eco-inefficiency score* ρ_t *of DEA problem (9) is exactly equal to the eco-inefficiency measure* ϖ_t *of LP-model (10). The dual solution of model (10) coincides with the solution of the DEA multiplier problem (which is the dual of problem (9)).*

Proof:
We start with the dual problem to (10).

$$\begin{aligned} \min \ & p'_{t,t}\, \mathrm{y}_t^0 + r'_{t,t}\, z_t^0 \\ \text{s.t.} \ & \\ & p'_{t,t}(I - A_t) + r'_{t,t} B_t \geq 0 \\ & -p'_{1;t,t}\, y_{1,t}^0 + r'_{t,t}\, z_t^0 = 1 \\ & p_{1;t,t} \leq 0,\ p_{2;t,t} \geq 0,\ r_{t,t} \geq 0 \end{aligned} \qquad (11)$$

where $p'_{t,t} = \left(p'_{1;t,t}, p'_{2;t,t}\right)$ with $p'_{1;t,t}$ the (1 × *n*) vector shadow prices of commodities, $p'_{2;t,t}$ the (1 × *o*) vector of shadow prices for abating pollutants, and $r'_{t,t}$ the (1 × *m*) vector of shadow prices of the primary inputs. Because of the indecomposability of $A_{11,t}$ it follows for the Leontief model that, for $y_t^0 \geq 0$, $x_t > 0$ and $(I - A_t)^{-1} > 0$ (cf. e.g. Nikaido 1968).

Multiplying the augmented Leontief inverse by $Q_{t,t}$ we obtain the gross production vectors augmented by the anti-pollution activity levels corresponding to the individually optimal outputs and primary inputs.

$$(I - A_t)^{-1} Q_{t,t} = T \geq 0$$

The total primary inputs required by maximized outputs are given by

$$B_t T = Z_{t,t}$$

The multiplier DEA model (dual model to (9)) is

$$\min \ u'_{t,t}\, \mathrm{y}_t^0 + v'_{t,t}\, z_t^0$$

[3] For the condition of the workability of Leontief systems and the notion of indecomposability of A_t see e.g. Nikaido (1968).

s.t.

$$\begin{aligned} &u'_{t,t}Q_{t,t}+v'_{t,t}Z_{t,t}\geq 0 \\ &-u'_{1;t,t}y^0_{1,t}+v'_{t,t}z^0_t=1 \\ &u_{1;t,t}\leq 0, u_{2;t,t}\geq 0, v_{t,t}\geq 0 \end{aligned} \tag{12}$$

where $u'_{t,t}=(u'_{1;t,t},u'_{2;t,t})$ with $u'_{1;t,t}$ the (1 × *n*) vector shadow prices of commodities, $u'_{2;t,t}$ the (1 × *o*) vector shadow prices for abating pollutants, and $v'_{t,t}$ the (1 × *m*) vector of (shadow) prices of the primary input factors.

Substituting $(I-A_t)T=Q_{t,t}$ and $B_tT=Z_{t,t}$ in the first constraint of (12) and multiplying the first constraint of (11) by yields

$$u'_{t,t}(I-A_t)T+v'_{t,t}B_tT\geq 0$$

and

$$p'_{t,t}(I-A_t)T+r'_{t,t}B_tT\geq 0$$

Obviously, the first constraints of the problems (12) and (11) are the same.

Since $p'_{t,t}\mathrm{y}^0_{\mathrm{t}}=u'_{t,t}y^0_t$ and $r'_{t,t}\mathrm{z}^0_{\mathrm{t}}=v'_{t,t}z^0_t$ the dual solutions coincide $p'_{t,t}=u'_{t,t}$, $r'_{t,t}=v'_{t,t}$ and the eco-inefficiency scores as well.

3 Eco-efficiency and eco-productivity change of the economy over time

The main aim of the paper is to analyse the eco-productivity change of an economy over time and to identify the main drivers of this change. For this purpose, following Chambers et al. (1996a, 1996b), we define non-oriented proportional Luenberger productivity indicator over two accounting periods (*t* and *t*+1) as:

$$\begin{aligned} PRODCH(z^0_t,y^0_{1,t},y^0_{2,t};z^0_{t+1},y^0_{1,t+1},y^0_{2,t+1}) = \frac{1}{2}\Big[&\big(\rho_{t+1}(z^0_t,y^0_{1,t},y^0_{2,t})-\rho_{t+1}(z^0_{t+1},y^0_{1,t+1},y^0_{2,t+1})\big) \\ +&\big(\rho_t(z^0_t,y^0_{1,t},y^0_{2,t})-\rho_t(z^0_{t+1},y^0_{1,t+1},y^0_{2,t+1})\big)\Big] \end{aligned} \tag{13}$$

Färe et al (2008) showed that in the special case of single output, constant returns to scale, and Hicks neutrality, the Luenberger productivity indicator is equivalent to the Solow specification of technical change. The non-oriented proportional Luenberger indicator of eco-productivity change can be decomposed into eco-efficiency change (catch-up, *EFFCH*) and eco-technical change (frontier shift, *TECHCH*) as follows:

$$EFFCH(z^0_t,y^0_{1,t},y^0_{2,t};z^0_{t+1},y^0_{1,t+1},y^0_{2,t+1})=\rho_t(z^0_t,y^0_{1,t},y^0_{2,t})-\rho_{t+1}(z^0_{t+1},y^0_{1,t+1},y^0_{2,t+1}) \tag{14}$$

$$\begin{aligned} TECHCH(z^0_t,y^0_{1,t},y^0_{2,t};z^0_{t+1},y^0_{1,t+1},y^0_{2,t+1}) = \frac{1}{2}\Big[&\big(\rho_{t+1}(z^0_{t+1},y^0_{1,t+1},y^0_{2,t+1})-\rho_t(z^0_{t+1},y^0_{1,t+1},y^0_{2,t+1})\big) \\ +&\big(\rho_{t+1}(z^0_t,y^0_{1,t},y^0_{2,t})-\rho_t(z^0_t,y^0_{1,t},y^0_{2,t})\big)\Big] \end{aligned} \tag{15}$$

This Luenberger indicator is expressed as the sum of *EFFCH* and *TECHCH*. *EFFCH* captures the average gain/loss in primary inputs and net outputs due to a difference in eco-inefficiency from period *t* to period *t*+1. *TECHCH* captures the average gain/loss in primary inputs and net outputs due to a shift in technology from period *t* to period *t*+1.

To compute the non-oriented proportional Luenberger indicator and its components, besides the estimation of two own-period eco-inefficiency scores, i.e. $\rho_t\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right)$ and $\rho_{t+1}\left(x_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right)$, we need the estimation of two cross-period eco-inefficiency scores:

1) $\rho_{t+1}\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right)$, which represents the degree of eco-inefficiency of an economy operating at *t* when evaluated with respect to the technology at *t*+1; and

2) $\rho_t\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right)$, which represents the degree of eco-inefficiency of an economy at *t*+1 when evaluated with respect to the technology at *t*.

First, the linear programming (LP) program in (9) is solved for two periods (*t* and *t*+1) to arrive at $\rho_t\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right)$ and $\rho_{t+1}\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right)$. For these LPs, separate output matrices Q_1, i.e. $Q_{1;t,t}$ and $Q_{1;t+1,t+1}$, separate pollution matrices Q_2, i.e. $Q_{2;t,t}$ and $Q_{2;t+1,t+1}$, and separate primary input matrices Z, i.e. $Z_{t,t}$ and $Z_{t+1,t+1}$, have to be constructed by solving the LPs (4) and (6) for each period.

Second, the cross-period distance function, $\rho_{t+1}\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right)$ can be set up as

$$
\begin{aligned}
\rho_{t+1}\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right) = \max_{\mu,\beta}\ & \beta \\
\text{s.t.}\ & \\
-\beta y_t^0 + Q_{1;t+1,t}\mu &\geq y_{1,t}^0 \\
Q_{2;t+1,t}\mu &\leq y_{2,t}^0 \\
\beta z_t^0 + Z_{t+1,t}\mu &\leq z_t^0 \\
\mu \geq 0,\ \beta\ &\text{free}
\end{aligned}
\qquad (16)
$$

Similarly, the other cross-period distance function, $\rho_t\left(x_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right)$ can be set up as

$$
\begin{aligned}
\rho_t\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) = \max_{\mu,\beta}\ & \beta \\
\text{s.t.}\ & \\
-\beta y_{t+1}^0 + Q_{1;t,t+1}\mu &\geq y_{1,t+1}^0 \\
Q_{2;t,t+1}\mu &\leq y_{2,t+1}^0 \\
\beta z_{t+1}^0 + Z_{t,t+1}\mu &\leq z_{t+1}^0 \\
\mu \geq 0,\ \beta\ &\text{free}
\end{aligned}
\qquad (17)
$$

For these two DEA models separate output matrices Q_1, i.e. $Q_{1;t+1,t}$ and $Q_{1;t,t+1}$, separate pollution matrices Q_2, i.e. $Q_{2;t+1,t}$ and $Q_{2;t,t+1}$, and separate primary input matrices Z, i.e. $Z_{t+1,t}$ and $Z_{t,t+1}$, have to be constructed by solving the LPs (4) and (6). For these computations, production technology on the one hand and available primary input as well as final demand and net pollution on the other are observed in different periods, which are indicated by the subscripts $t+1,t$ and $t,t+1$. In total, the LPs (4) and (6) have to be solved four times.

In the case of the cross-period LP programs, $\rho_{t+1}\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right)$ in (16) and $\rho_t\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right)$ in (17), when the economy under evaluation remains outside the technology set it is considered 'super eco-efficient', meaning the eco-inefficiency score β becomes negative. Such an eco-inefficiency score implies that the primary inputs need to be increased and net outputs need to be decreased to get such super eco-efficient economies projected onto the eco-efficiency frontier.

The proposed method allows the researcher to examine the reasons of *EFFCH*, *TECHCH* and eco-productivity change (*PRODCH*). It attributes the use of individual primary input as well as individual commodity and individual pollutant to eco-productivity change and its components. To show this we start first by deriving the formula for *EFFCH*, before we present the formulae for *TECHCH* and *PRODCH*.

The starting points of this analysis are the definition (formula (14)) change and the dual to the DEA model (9) as it is shown in model (12). After a few steps of remodelling (see appendix) it turns out that the contribution of the *i*-th primary input is

$$EFFCH_i = v_{i;t,t} z_{i;t}^0 - v_{i;t+1,t+1} z_{i;t+1}^0 \tag{18}$$

that of the *j*-th commodity

$$EFFCH_j = u_{1,j;t,t} y_{1,j;t}^0 - u_{1,j;t+1,t+1} y_{1,j;t+1}^0 \tag{19}$$

and that of the *k*-th pollutant

$$EFFCH_k = u_{2,k;t,t} y_{2,k;t}^0 - u_{2,k;t+1,t+1} y_{2,k;t+1}^0 \tag{20}$$

The aggregate indicator for *EFFCH* can then be arrived at by summing up their respective input-specific indicators, the respective commodity specific indicators and the respective pollutant specific indicators (i.e. $EFFCH = \sum_{i=1}^{m} EFFCH_i + \sum_{j=1}^{n} EFFCH_j + \sum_{k=1}^{o} EFFCH_k$).

For *TECHCH*, our starting points are the definition (formula (15)) and the duals to the DEA models (9), (16) and (17). As a result, the contribution of the *i*-th primary input is given by

$$TECHCH_i = \frac{1}{2}\left(v_{i;t+1,t+1} z_{i;t+1}^0 - v_{i;t,t+1} z_{i,t+1}^0 + v_{i;t+1,t} z_{i;t}^0 - v_{i;t,t} z_{i;t}^0\right) \tag{21}$$

that of the *j*-th commodity

$$TECHCH_j = \frac{1}{2}\left(u_{1,j;t+1,t+1}y^0_{1,j;t+1} - u_{1,j;t,t+1}y^0_{1,j;t+1} + u_{1,j;t+1,t}y^0_{1,j;t} - u_{1,j;t,t}y^0_{1,j;t}\right) \tag{22}$$

and that of the *k*-th polutant

$$TECHCH_k = \frac{1}{2}\left(u_{2,k;t+1,t+1}y^0_{2,j;t+1} - u_{2,k;t,t+1}y^0_{2,k;t+1} + u_{2,k;t+1,t}y^0_{2,k;t} - u_{2,k;t,t}y^0_{2,k;t}\right) \tag{23}$$

Again, the aggregate indicator of *TECHCH* can be achieved by summing up the contributions of all primary inputs, all commodities and all pollutants (i.e. $TECHCH = \sum_{i=1}^{m} TECHCH_i + \sum_{j=1}^{n} TECHCH_j + \sum_{k=1}^{o} TECHCH_k$).

For *PRODCH* we begin with the definition (formula (13)) and the duals to the DEA models (9), (16) and (17). Hence, the contribution of the *i*-th primary input is given by

$$PRODCH_i = \frac{1}{2}\left(v_{i;t+1,t}z^0_{i;t} - v_{i,t+1,t+1}z^0_{i;t+1} + v_{i,t,t}z^0_{i;t} - v_{i,t,t+1}z^0_{i;t+1}\right) \tag{24}$$

that of the *j*-th commodity by

$$PRODCH_j = \frac{1}{2}\left(u_{1,j;t+1,t}y^0_{1,j;t} - u_{1,j;t+1,t+1}y^0_{1,j;t+1} + u_{1,j;t,t}y^0_{1,j;t} - u_{1,j;t,t+1}y^0_{1,j;t+1}\right) \tag{25}$$

and that of the *k*-th pollutant by

$$PRODCH_k = \frac{1}{2}\left(u_{2,k;t+1,t}y^0_{2,j;t} - u_{2,k;t+1,t+1}y^0_{2,j;t+1} + u_{2,k;t,t}y^0_{2,k;t} - u_{2,k;t,t+1}y^0_{2,k;t+1}\right) \tag{26}$$

The sum of the contributions of all primary inputs, all commodities and all pollutants is exactly equal to *PRODCH* (i.e. $PRODCH = \sum_{i=1}^{m} PRODCH_i + \sum_{j=1}^{n} PRODCH_j + \sum_{k=1}^{o} PRODCH_k$).

It can be shown that the contribution of the *i*-th primary input to *EFFCH* plus the contribution of the *i*-th primary input to *TECHCH* is equal to the contribution of the *i*-th primary input to *PRODCH* (i.e. $PRODCH_i = EFFCH_i + TECHCH_i$). Furthermore, the contribution of the *j*-th commodity to *EFFCH* plus the contribution of the *j*-th commodity to *TECHCH* is equal to the contribution of the *j*-th commodity to *PRODCH* (i.e. $PRODCH_j = EFFCH_j + TECHCH_j$). Finally, the contribution of the *k*-th pollutant to *EFFCH* plus the contribution of the *k*-th pollutant to *TECHCH* is equal to the contribution of the *k*-th pollutant to *PRODCH* (i.e. $PRODCH_k = EFFCH_k + TECHCH_k$).

This decomposition enables the researcher to empirically examine the contributions of each individual primary input, individual commodity and individual pollutant towards the eco-productivity change and its components – eco-efficiency change and eco-technical change. Since the same inefficiency measure β is used for adjusting inputs as well as desirable outputs the following proposition can be proved.

Proposition 2: *The total contribution of the primary inputs* z^0 *is equal to the total contribution of all commodities (good outputs)* y_1^0. *This holds for PRODCH as well as for both components EFFCH and TECHCH.*

We show this for *EFFCH*. For *PRODCH* and *TECHCH* the relationship can be shown in an analogue way.

$$\begin{aligned} EFFCH &= \rho_t\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right) - \rho_{t+1}\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) = \\ &= \left(u_{1;t,t}y_{1,t}^0 + u_{2;t,t}y_{2,t}^0 + v_{t,t}z_t^0\right) - \left(u_{1;t+1,t+1}y_{1,t+1}^0 + u_{2;t+1,t+1}y_{2,t+1}^0 + v_{t+1,t+1}z_{t+1}^0\right) = \\ &= v_{t,t}z_t^0 - v_{t+1,t+1}z_{t+1}^0 + u_{1;t,t}y_{1,t}^0 - u_{1;t+1,t+1}y_{1,t+1}^0 + u_{2;t,t}y_{2,t}^0 - u_{2;t+1,t+1}y_{2,t+1}^0 \end{aligned} \qquad (27)$$

$v_{t,t}z_t^0 - v_{t+1,t+1}z_{t+1}^0$ is the total contribution of all primary inputs and $u_{1;t,t}y_{1;t}^0 - u_{1;t+1,t+1}y_{1;t+1}^0$ the total contribution of all commodities and $u_{2;t,t}y_{2;t}^0 - u_{2;t+1,t+1}y_{2;t+1}^0$ the total contribution of all pollutants. It has to be shown that $v_{t,t}z_t^0 - v_{t+1,t+1}z_{t+1}^0 = u_{1;t,t}y_{1;t}^0 - u_{1;t+1,t+1}y_{1;t+1}^0$. After a short transformation of (27) we obtain $-u_{1;t,t}y_{1,t}^0 + v_{t,t}z_t^0 = -u_{1;t+1,t+1}y_{1,t+1}^0 + v_{t+1,t+1}z_{t+1}^0$. From (12) we know that $-u_{t,t}y_{1,t}^0 + v_{t,t}z_t^0 = 1$ and $-u_{t+1,t+1}y_{1,t+1}^0 + v_{t+1,t+1}z_{t+1}^0 = 1$ which complete the proof.

4 An Illustrative Empirical Application

In this section, we describe how our model can be used to estimate eco-productivity growth in Austria, in order to demonstrate the applicability of our proposed approach. In order to investigate their meaning for eco-productivity growth, we compute the contributions of different primary inputs, pollution (undesirable output) and individual commodities (desirable output).

4.1 Dataset

Our data set comprises the two most important primary inputs: labour and capital. *Labour* is subdivided into three levels: high-skilled, medium-skilled and low-skilled. These are classified according to the International Standard Classification of Education (ISCED). Thus, high-skilled labour is defined as workers who completed the first or also second phase of tertiary education (levels 5-6); medium-skilled as workers who completed upper secondary education or postsecondary not tertiary education (levels 3-4) and low-skilled as workers who completed lower secondary education or less (levels 0-2)[4]. The data source of labour used is Socio-Economic Accounts of the World Input-Output Database (WIOD). Labour is measured in millions of hours worked. The time series were downloaded in July 2012. *Capital* is represented by total net fixed assets at replacement prices of 2005 and contains all asset types - including crop plants, kits, vehicles, residential buildings, other types of buildings, intangible assets. It is taken from Statistics

[4] The end of level 2 often coincides with the end of compulsory schooling.

Austria and measured in billions of EUR. The capital stock actually used cannot be observed directly and has to be estimated. This estimation is done by multiplying data on fixed capital stock taken from Statistics Austria[5] by capacity utilization rate for total industry taken from Eurostat. Hence, our empirical model consists of four primary inputs: labour of three skill types, and capital. These data serve as a basis for computation of the matrices of primary input requirement per unit of gross output for production activities (B_1-matrices) of the respective years. In our empirical application we cover an observation period from 1995 to 2007.

Since our model is based on an augmented Leontief model data on *environmental workers* are required. As these data are not available from any data source they have to be estimated. In a first step we compute the share of labour compensation paid for environmental protection (section ambient air and climate) on total labour compensation for industry and construction sectors. In a second step we multiply this share with the total number of employees to get the estimated total number of environmental workers. Finally, we distribute the total number to the three levels of three skill-levels and converte in million hours worked. Data on labour compensation and total number of employees are taken from Austrian Economic Chamber (1998, 1999) and Statistics Austria (2008a, 2008b). In addition, data on *environmental capital stock* are required. These data are not available from any data source and has to be estimated as well. This estimation is done by applying the perpetual inventory method based on time series of gross fixed capital formation for environmental protection from Austrian Economic Chamber (until the year 2002) and Statistics Austria (from the year 2002 on). These data are used for computation of the matrix of primary input coefficients for abatement activities (B_2-matrices) of the respective years.

The interrelationship between the industries is measured by input-output tables that are based on domestic use tables as well as make tables. The input-output tables of Austria for 1995 (Statistics Austria 2001) and 2007 (Statistics Austria 2011a) are deflated to price level of 2005 by applying the approach developed by Dietzenbacher and Hoen (1998) and Koller and Stehrer (2010) in order to exclude changes of the A-matrices due to the change of relative prices. Because our analyses are done mainly for illustration purposes we content ourselves with rather highly aggregated input-output tables. The tables are aggregated to eighteen times eighteen[6] by applying the approach of Olsen (2000 and 2001) (see also Kymn 1990; Kymn and Norsworthy 1976 for comprehensive surveys of aggregation approaches). From these input-output tables the conventional domestic input coefficient matrices (A_{11}-matrices) are computed.

We augment the conventional input-output tables by adding *pollution abatement for climate protection and pollution control* and *air emissions* (sum of SO_2, NO_X, NMVOC, CH_4, CO, CO_2, N_2O, NH_3 and PM_{10}). These data were obtained from integrated NAMEA (National Accounting Matrix including Environmental Accounts). For a detailed description see Statistics Austria (2011b). Based on these data we compute the matrix of inputs per unit of eliminated pollutant (A_{12}-matrices) and the matrix showing the output of pollutant per unit of good (A_{21}-matrices). Since we take just one type of pollution (i.e. air emissions) in to account the respective matrices are one

5 We consider the fixed capital stock provided by Statistics Austria as capital endowment and not as capital actually used.

6 The input-output tables are aggregated in accordance with the structure of the pollution abatement and air emission data provided by Statistics Austria.

column vector (A_{12}-matrices) and one row vector (A_{21}-matrices). The matrix showing the output of pollutant per unit of eliminated pollutant (A_{22}-matrices) is just a zero in our application.

Final demand for economic commodities (y_1-vectors) serves as the measure of desirable outputs and consists of eighteen aggregates of commodities. These data are taken from Statistics Austria (2001) and Statistics Austria (2011a). The tolerated level of net pollution (y_2-vectors), which is our measure of undesirable output, is specified as reduction of air emission by 10% of the 1995 level of gross emissions (in the year 1995) and as a decline by 30% of the 1995 level of gross emissions (in the year 2007).

The vector of *gross industrial outputs* (x_1-vectors) serving as a basis of the computation of the input requirement matrix of production activity (B_1-matrices) is taken from the input-output tables. The *anti-pollution activity level* (x_2-vectors) serving as a basis of the computation of the matrix of primary input coefficients for abatement activities (B_2-matrices) is calculated as the difference of the gross pollution level of the respective year minus the tolerated level of net pollution (y_2-vectors). Since we take into account just one pollutant (i.e. air emissions) the anti-pollution activity level is just a scalar.

The labour endowment in hours of the Austrian economy cannot be observed directly. Therefore, we have to estimate them by applying the following procedure. In a first step, we take data on the population of different skill levels of the respective years from Statistics Austria (for the year 1995) and Eurostat (for the year 2007). We define the working-age population as all persons between 15 and 64 years. These data are given in number of persons. The working-age population cannot be considered as labour endowment in the sense of our model (labour component of the z^0-vectors) since it does not measure the economically active population or labour force[7]. In order to get the right measure of labour supply we multiply the working-age population with the labour force participation rate of different skill levels taken from Eurostat. Since our labour used in production is measured in hours worked the labour supply data given in number of persons need to be converted to get suitable data on labour endowment of the Austrian economy. We multiply the number of person by the number of hours usually worked by a full time employed person during a year[8]. The capital stock endowment data (capital component of the z^0-vectors) are taken from Statistics Austria. Final demand data (together with the input-output tables and cross industrial outputs) are taken from Statistics Austria (2000 and 2011a). We deflate them by applying the approach developed by Dietzenbacher and Hoen (1998) and Koller and Stehrer (2010).

7 According to the definition of United Nations SNA system of national accounts (United Nations, 2009) the economically active population or labour force comprises all persons who supply labour for the production of economic goods and services, during a specified time reference period. It equals the sum of the employed and the unemployed.

8 In Austria these are normally 1785.18 hours a year. Following Ortner and Ortner (2012, p. 192) we calculate this as follows: The monthly working hours for a full time employed worker are 173.00 hours (§3 Abs. 1 AZG). We multiply the monthly working hours by 12 months to get the yearly working hours of 2076.00 hours. From this we subtract hours of not performance times due to vacations and due to legal holidays. The number of vacation days are 25 working days (§2 Abs. 1 UrlG) and the number of legal holidays are 12 days (§7 Abs. 2 ARG). The number of vacation days and the number of legal holidays are multiplied by 7.86 hours (Ortner and Ortner, 2012, p. 734). The computation in detail is the following: 2076.00 – (25 + 12) * 7.86 = 1785.18.

4.2 Descriptive statistics

This section presents some descriptive statistics. They give a first impression of the eco-productivity development of Austria during the observation period and support the interpretation of the estimation results presented in the next section.

Tab. 1: Utilization of primary inputs

	resources used (1)	endowment (2)	utilization (=(1)/(2))
		1995	
High-skilled labour (in millions hours)	741	767	0.97
Medium-skilled labour (in millions hours)	3,974	4,219	0.94
Low-skilled labour (in millions hours)	1,435	1,792	0.80
Capital, all assets (in billions EUR)	597	708	0.84
		2007	
High-skilled labour (in millions hours)	1,231	1,298	0.95
Medium-skilled labour (in millions hours)	4,305	4,687	0.92
Low-skilled labour (in millions hours)	1,255	1,421	0.88
Capital, all assets (in billions EUR)	811	915	0.89

Table 1 shows data and utilization rates of primary inputs. For high and medium-skilled labour as well as capital, the quantities used and the endowments clearly increased. For low-skilled labour, quantities used and endowment clearly decreased. Furthermore, from Table 1 it can be seen that the resource utilization (i.e. ratio of resources used to resource endowment) of high-skilled and medium-skilled labour worsened, whereas of low-skilled labour as well as of capital stock improved. These data indicate that the utilization of the two scarcest resources decreased. Based on this development it can be expected that the eco-efficiency change component of the Luenberger indicator will reveal eco-efficiency regress of the whole economy.

Tab. 2: Descriptive statistics of final demand and pollution

	1995	2007	growth rate
	in billions EUR		in percent
Products of agriculture, hunting, forestry and fishing	1.15	1.64	42.48
Mining	0.21	0.65	210.14
Food, beverages and tobacco	8.82	10.98	24.54
Textiles and leather	3.39	2.84	-16.16
Wood and products of wood	2.12	4.36	105.53
Paper and printed matter	3.46	5.90	70.40
Chemical and refined petroleum products	4.84	9.18	89.74
Other non-metallic mineral products	2.29	2.44	6.59
Basic metals	4.11	9.47	130.71
Machinery and equipment	18.03	31.52	74.76
Motor vehicles and transport equipment	5.74	16.46	186.89
Other manufactured goods	7.27	10.08	38.73
Electrical Energy	2.21	4.28	93.35
Construction Work	21.86	23.12	5.79
Land transport services	5.35	6.16	15.23
Water transport services	0.07	0.06	-17.05
Air transport services	0.65	2.10	225.16
Other services and public administration	126.59	174.01	37.46
Total	**218.14**	**315.25**	**44.51**
	in millions tons		in percent
Net pollution (i.e. net air emissions)	**44.66**	**34.73**	**-22.22**

Table 2 presents the data on final demand, tolerated level of net pollution and their growth. It can be seen that the final demand for almost all commodities increased, and growth rate differs from commodity to commodity. The pollution (net air emissions) clearly decreased due to an active environmental policy.

Tab. 3: Structure of final demand

	1995	2007	change
	in percent		in percentage points
Products of agriculture, hunting, forestry and fishing	0.53	0.52	-0.01
Mining	0.10	0.21	0.11
Food, beverages and tobacco	4.04	3.48	-0.56
Textiles and leather	1.55	0.90	-0.65
Wood and products of wood	0.97	1.38	0.41
Paper and printed matter	1.59	1.87	0.28
Chemical and refined petroleum products	2.22	2.91	0.69
Other non-metallic mineral products	1.05	0.77	-0.28
Basic metals	1.88	3.00	1.12
Machinery and equipment	8.27	10.00	1.73
Motor vehicles and transport equipment	2.63	5.22	2.59
Other manufactured goods	3.33	3.20	-0.13
Electrical Energy	1.01	1.36	0.34
Construction Work	10.02	7.33	-2.68
Land transport services	2.45	1.95	-0.50
Water transport services	0.03	0.02	-0.01
Air transport services	0.30	0.67	0.37
Other services and public administration	58.03	55.20	-2.83

Table 3 compares structure of final demand in 1995 and 2007. In terms of change in share the commodity Motor vehicles and transport equipment gained remarkable whereas the commodities construction work as well as other services and public administration clearly shrank. Overall the final demand for commodities of the secondary sector increased more than of the agricultural sector and the service sector.

Tab. 4: Primary input requirement matrices (B-matrices)

	primary sector	secondary sector	tertiary sector	**mean**	pollution abatement
			1995		
high-skilled labour [1]	61.25	12.10	1.73	**1.47**	0.05
medium-skilled labour [1]	91.83	109.47	10.73	**13.57**	0.08
low-skilled labour [1]	61.25	39.19	3.19	**6.29**	0.03
capital total [2]	6,35	10.65	1.81	**1,35**	0.79
			2007		
high-skilled labour [1]	9.81	15.98	1.55	**1.78**	0.13
medium-skilled labour [1]	48.98	69.07	8.17	**8.38**	0.21
low-skilled labour [1]	22.24	19.97	2.09	**2.81**	0.08
capital total [2]	5.08	7.85	1.68	**1.09**	0.25

Notes: [1] in hours worked per 1.000 EUR produced (labour per cross production)
[2] in unity (capital per cross production)

Table 4 presents the aggregated primary input requirement matrices of 1995 and 2007. Input requirement is defined as the ratio of amount of primary inputs used in a sector divided by cross output produced of a sector and tells how much of a resource is needed to produce one unit of gross output. A decrease over time indicates an increase of the productivity of this factor in the respective sector. In such a case, fewer resources are required to produce one unit of output.

From this table it can be seen that in most sectors, as well as on average, the values of primary input requirements for production activities decreased. The development for pollution abatement was clearly different from that of production activities. The labour requirement per unit of prevented emissions increased very much whereas the capital requirement decreased. In spite of the decrease of labour productivity of the abatement activities it can be expected that overall the eco-technical change component of the Luenberger Indicator will indicate eco-technical progress since pollution abatement accounts for a relatively small share of activities of the Austrian economy. From the estimation results of the contributions to eco-technical change we probably will see that pollution abatement contributes negatively.

Tab. 5: Emission coefficients 1995 and 2007

		primary sector	Secondary sector	Tertiary sector	**overall**
1995	in tons per	0.360	0.310	0.041	**0.152**
2007	1,000 EUR	0.304	0.275	0.038	**0.138**

Note: weighted averages of commodities

Table 5 shows the emission coefficients of 1995 and 2007. This indicator measures how much air emissions are generated per unit of gross production. A decrease of these coefficients indicates environmental saving technical progress. Such a development can be seen from the numbers in Table 5. The values clearly decreased in all three sectors as well as overall. This data indicate a positive contribution of abatement activities to eco-technology. The net effect of the developments shown in Table 4 and Table 5 is a priori unclear and can be seen only from estimations by means of our model.

4.3 Estimation results

First we compute the eco-inefficiency scores and the shadow prices for the years 1995 and 2007 applying the DEA model [(9) and (12)] and the Leontief model [(10) and (11)] in order to show empirically that the estimation results of both models are equal. As can be seen from Table 6, this is indeed the case, and the empirical results confirm the statement of proposition 1. The first line below the column heading shows eco-inefficiency scores of 0.0168 and 0.0263 in the years 1995 and 2007, respectively. These results can be interpreted as follows: in both years the Austrian economy is eco-inefficient, in the sense that its actual performance deviates from its potential and its resources are not fully utilized. In 1995 the Austrian economy could increase its actual final demand and decrease the actual use of primary inputs by round 1.7 percent, simultaneously. In 2007, the Austrian economy is even more eco-inefficient, further away from its possibilities, and its potential for improvement is larger than in 1995. It could raise the actual final demand and reduce the primary inputs actually used by around 2.6 percent.

Tab. 6: Eco-inefficiency scores and shadow prices from single period DEA and augmented Leontief model for 1995 and 2007

		1995		2007	
		DEA-model	Leontief-model	DEA-model	Leontief-model
Eco-inefficiency scores		0.0167	0.0167	0.0262	0.0262
		shadow prices			
Final demand	Products of agriculture, hunting, forestry and fishing	-0.00564	-0.00564	-0.00596	-0.00596
	Mining	-0.00158	-0.00158	-0.00113	-0.00113
	Food, beverages and tobacco	-0.00262	-0.00262	-0.00202	-0.00202
	Textiles and leather	-0.00127	-0.00127	-0.00125	-0.00125
	Wood and products of wood	-0.00238	-0.00238	-0.00185	-0.00185
	Paper and printed matter	-0.00164	-0.00164	-0.00109	-0.00109
	Chemical and refined petroleum products	-0.00107	-0.00107	-0.00063	-0.00063
	Other non-metallic mineral products	-0.00185	-0.00185	-0.00126	-0.00126
	Basic metals	-0.00129	-0.00129	-0.00067	-0.00067
	Machinery and equipment	-0.00150	-0.00150	-0.00098	-0.00098
	Motor vehicles and transport equipment	-0.00099	-0.00099	-0.00055	-0.00055
	Other manufactured goods	-0.00158	-0.00158	-0.00126	-0.00126
	Electrical Energy	-0.00180	-0.00180	-0.00094	-0.00094
	Construction Work	-0.00109	-0.00109	-0.00126	-0.00126
	Land transport services	-0.00210	-0.00210	-0.00144	-0.00144
	Water transport services	-0.00192	-0.00192	-0.00101	-0.00101
	Air transport services	-0.00181	-0.00181	-0.00079	-0.00079
	Other services and public administration	-0.00277	-0.00277	-0.00187	-0.00187
Pollution	Air emissions	0.00015	0.00015	0.00004	0.00004
Primary inputs	Low-skilled labour	0	0	0	0
	Medium-skilled labour	0	0	0	0
	High-skilled labour	0.00066	0.00066	0.00039	0.00039
	Capital	0	0	0	0

Note: DEA model … models (9) and (12), Leontief model … models (10) and (11)

Additionally, Table 6 shows the results of shadow prices computations. Positive shadow prices of primary inputs indicate that an increase in the endowment ceteris paribus raises eco-inefficiency because of the increased difference between the endowment and utilized quantities. This is also true for net pollution. An increase in the tolerated level of air emissions (undesirable output) increases eco-inefficiency. Conversely, negative shadow prices of commodities reveal that an increase in final demand (desirable output) ceteris paribus reduces eco-inefficiency. Generally speaking, a non-zero shadow price indicates that the respective resource or commodity is scarce, and the environmental standard for the respective pollutant is restrictive. By contrast, a value of zero implies that a change in endowment or final demand or net pollution does not change eco-inefficiency and shows the respective resource or commodity is abundant or the environmental standard for the respective pollutant is not restrictive. The results presented in Table 6 indicate clearly that in 1995 as well as in 2007 high-skilled labour is scarce, whereas the other primary inputs are abundant. Thus, only one primary input is scarce. This follows from the assumption of the linear programming technology without direct substitution possibilities where the value of the objective function (inefficiency score) is determined by the scarcest resource. An additional unit of high-skilled labour (with all other things held constant) raises eco-inefficiency, whereas an increase of all other primary inputs does not change eco-inefficiency. From 1995 to 2007 the shadow price of high-skilled labour decreases clearly indicating that this type of labour becomes less scarce. This development corresponds to the decrease of utilization of high-skilled labour shown in Table 1. A similar situation can be found for net air emissions (undesirable output). The shadow price is positive indicating that an increase of tolerated level of pollution raises eco-inefficiency. Furthermore, according to the shadow prices listed in Table 6 an increase in final demand (desirable output) for any commodity decreases eco-inefficiency in both years indicating all of them are scarce.

Tab. 7: Results of Luenberger Indicator and its components, 1995 to 2007

		DEA model	Leontief model
Eco-inefficiency scores	in 1995	0.0167	0.0167
	in 2007	0.0262	0.0262
	1995 to 2007 (mixed period)	-0.1747	-0.1747
	2007 to 1995 (mixed period)	-0.0702	-0.0702
Luenberger Indicator	Eco-efficiency change (*EFFCH*)	**-0.0096**	**-0.0096**
	Eco-technical change (*TECHCH*)	**0.0570**	**0.0570**
	Eco-productivity change (*PRODCH*)	**0.0474**	**0.0474**

In a next step, we apply our DEA models and the Luenberger productivity indicator to estimate the eco-productivity change (*PRODCH*) in the Austrian economy from 1995 to 2007. The results are shown in Table 7. Again, we apply our DEA model as well as the augmented Leontief model, and compare the results in order to check whether the outcomes coincide. It turns out that the results of both models are exactly the same. The eco-inefficiency scores of single period and mixed period models are equal. As a consequence, the values of *EFFCH*, *TECHCH* and *PRODCH* are the same as well. According to our results, the eco-efficiency of the Austrian economy slightly decreases. The *EFFCH* score is around minus 1 indicating eco-efficiency regress of approximately 1 percent. This outcome confirms our expectations drawn from Table 1 and is plausible against the background of a general increase of unemployment in Austria during the observation period. Contrary to eco-efficiency regress, we find a positive *TECHCH* score of around 5.7 indicating technical progress of 5.7 percent. This value shows that the Austrian economy goes through a clear eco-technical progress during the observation period. This result confirms our expectations drawn from Table 4. By means of the *EFFCH* scores and *TECHCH* scores *PRODCH* can be easily computed as *PRODCH* is simply equal to the sum of *EFFCH* and *TECHCH*. This simple procedure yields a PRODCH score of around 4.7 indicating eco-productivity progress of circa 4.7 percent.

These results raise the question as to which primary input as well as which commodity cause *EFFCH*, *TECHCH* and *PRODCH* and what is the role of pollution. Or in other words, what are the contributions of the individual inputs as well as desirable and undesirable outputs to eco-efficiency, eco-technology and eco-productivity development. To answer these questions, we apply the approach described in the previous section, i.e. the formulae (18) to (26), which combines shadow prices and observed data (endowment of primary input as well as exogenous given final demand for commodities and tolerated level of air emissions) to estimate the contributions of individual primary inputs as well as desirable and undesirable outputs. The results are shown in Table 8. Note that the column sums are equal to the *EFFCH* score, *TECHCH* score and *PRODCH* score, respectively.

Tab. 8: contribution of each individual output and primary input

		Eco-efficiency change (*EFFCH*)	Eco-technical change (*TECHCH*)	Eco-productivity change (*PRODCH*)
Final demand	Products of agriculture, hunting, forestry and fishing	0.0033	0.0085	0.0118
	Mining	0.0004	0.0001	0.0005
	Food, beverages and tobacco	-0.0009	0.0230	0.0222
	Textiles and leather	-0.0008	0.0003	-0.0005
	Wood and products of wood	0.0030	0.0042	0.0072
	Paper and printed matter	0.0008	0.0022	0.0030
	Chemical and refined petroleum products	0.0006	0.0021	0.0026
	Other non-metallic mineral products	-0.0012	0.0007	-0.0004
	Basic metals	0.0010	0.0037	0.0047
	Machinery and equipment	0.0039	0.0087	0.0126
	Motor vehicles and transport equipment	0.0034	0.0039	0.0074
	Other manufactured goods	0.0012	0.0033	0.0044
	Electrical Energy	0.0000	0.0005	0.0005
	Construction Work	0.0053	-0.0013	0.0041
	Land transport services	-0.0024	0.0017	-0.0007
	Water transport services	-0.0001	0.0000	0.0000
	Air transport services	0.0005	0.0009	0.0014
	Other services and public administration	-0.0256	-0.0313	-0.0569
pollution	Air emissions	0.0054	-0.0055	-0.0001
primary inputs	low-skilled labour	0	-0.2049	-0.2049
	medium-skilled labour	0	0	0
	high-skilled labour	-0.0075	0.2362	0.2287
	capital	0	0	0
		-0.0096	**0.0570**	**0.0474**

It turns out that eco-efficiency regress is driven by a decline in utilization of certain primary inputs as well as insufficient growth of the final demand for certain commodities. Among the primary inputs high-skilled workers' contribution to EFFCH is clearly negative. This corresponds to the findings of Table 1 showing a decrease in utilization of this resource. Due to the linearity of the model, the scarcest resource (i.e. high-skilled labour in our case) determines eco-inefficiency scores and therefore eco-efficiency change results and contributions. The results also reflect the change in terms of shortage, which can be seen from shadow prices in Table 6. The shadow price of high-skilled labour decreases from 1995 to 2007 but the endowment increases and consequently the expenditures increases. Since the contribution of a primary input is the difference between expenditures in the first year and in the second year (see formula (18)), the contribution turns out to be negative. The contributions of the other three primary inputs amount to zero as the shadow prices of all of them are equal to zero in both years.

For net air emission Table 8 shows a positive contribution to eco-efficiency change. In Table 2 we see a decrease of quantity from 1995 to 2007 and in Table 6 a decline of shadow prices. Both induce lower expenditures in the second year and consequently increase eco-efficiency. Our model indicates that a decrease of tolerated level of air emissions (i.e. improvement of environmental standards) improves eco-efficiency.

The contributions of twelve out of eighteen commodities are small and positive. For all of these commodities the economic revenue is higher in the first year than in the second year. For almost all of these commodities the growth of final demand is above average (see Table 2) and overcompensats the decrease of shadow prices. For the commodities which contribute negatively the growth of final demand is below average. The commodity called other services and public administration shows the highest negative contribution since its final demand is the largest of all negative contributing commodities.

Eco-technical progress in the Austrian economy is driven by primary inputs as well as desirable and undesirable outputs. Among the primary inputs, high-skilled labour contributes clearly positively whereas low-skilled labour contributes clearly negatively. The positive contribution of the former exceeds the negative one of the later so as to totally the contribution of labour is positive. This outcome corresponds to the findings of Table 4 showing that the production technology becomes more skill-intensive (i.e. more requirement of high-skilled and less requirement of low-skilled labour). The relationship between the contributions of high-skilled and low-skilled labour reflects the well-known substitution between these two production factors. In our model the substation between production factors is just indirect caused by the change in final demand and the subsequent change in gross production. Medium-skilled labour and capital do not contribute since its shadow prices are always zero, showing that they are not scarce resources neither in the first year nor in the second. The contributions of final demand for any commodity are marginally positive, with the exception of the clearly negative contribution of other services and public administration. The compulsory reduction of tolerated level of net air emissions contributes negatively to eco-technical change although environmental conditions improve. This is caused by an increase in resource intensity of pollution abatement activity (especially in terms of labour) which can be seen from Table 4. More resources are necessary for the same amount of abated pollution which reflects diminishing marginal return of abatement. Obviously, the increase in resource requirement outbalances the improvement in emission coefficients shown in Table 5.

In Table 8 the rightmost column shows the contribution to eco-productivity progress. The contribution to *PRODCH* is exactly the row sum, and thus, the sum of the contributions of individual factors to eco-efficiency change and eco-technical change of the respective commodity, net pollution and primary input. The total contributions of primary inputs and final demand to *PRODCH* are clearly positive. High-skilled labour contributes positively, while low-skilled labour negatively. The contributions of two contributing primary inputs are large relative to that of individual commodities and net pollution, whose contributions are small but mostly positive. The only exception is the commodity other services and public administration with a distinct negative contribution. The reduction of emissions (induced by environmental policy) seems to be almost neutral for the eco-productivity development in Austria.

5 Conclusions

In this paper we have sought to extend the existing literature on eco-efficiency methods by proposing a model that represents a given economy by way of an augmented Leontief input-output model extended by constraints of primary inputs. Using multi-objective optimization models an eco-efficiency frontier of the economy is generated. Its solutions define eco-efficient virtual decision making units (DMUs). The eco-efficiency of a given economy is defined as the difference between the potential of an economy and its actual performance and can be obtained as by using a data envelopment analysis (DEA) model. This model considers inputs, desirable outputs and undesirable outputs simultaneously taking into account both economical and ecological aspects. A Luenberger productivity indicator is proposed to estimate eco-productivity change over time; this is then decomposed in a way that enables one to examine the contributions of individual production factors, individual pollutants and individual commodties to eco-productivity change. The results allow an inference as to whether economic growth, primary input-saving or environmental-saving are the underlying drivers of eco-productivity change. Eco-productivity change (*PRODCH*), in turn, is decomposed into eco-efficiency change (*EFFCH*, catch-up) and eco-technical change (*TECHCH*, frontier shift) components.

This method has several advantages. Firstly, it allows us to take into account the interdependences of the sectors in an economy. Secondly, it permits estimating the eco-performance of an economy with respect to its own potential and without any comparison with other economies – economies that may possess different technologies and varying mutual interdependencies due to international trade. Finally, it integrates the materials balance principle[9] into the measurement of eco-productivity change. Though the theoretical foundations of this principle have been worked out by several authors (see e.g. Coelli et al., 2007; Ebert and Welsch, 2007; Førsund, 2009; Pethig, 2006), models usable for applied research are very rare. The present paper aims to fill in this gap in the literature by operationalizing this principle to an empirically implementable form and extending it to an intertemporal setting suitable to analyze eco-productivity change over time.

For illustration purposes this approach is used to analyze the long-term performance of the Austrian economy for the period 1995–2007. In order to account for pollution and abatement activities emissions to the air and expenditure on climate protection are included in the model. As a result, a clear overall eco-productivity growth of 4.8 percent is observed, with primary inputs and final demand each contributing 2.4 percent confirming the statement of proposition 2. Whereas air emissions contribute almost zero indicating that its impact is almost neutral. Therefore, it is concluded that productivity growth is driven by economic growth as well as by primary input-saving, and not by environmental-saving. Among the primary inputs considered in this example, high-skilled labour shows positive contribution to eco-productivity growth and low-skilled labour negative one while medium-skilled labour and capital contribute neither positively nor negatively. Of all commodities, food and beverages together with machinery and equipment contribute most positively to eco-productivity growth and other services and public administration most negatively. A closer look at eco-efficiency change and eco-technical change components reveals eco-technical progress as the main driver of eco-productivity progress. Eco-technical progress is mostly driven by high-skilled labour whereas low-skilled labour contributes negatively. Conversely, high-skilled labour contributes negatively to eco-efficiency change caused by the less efficient usage of this resource in 2007 as compared to 1995 owing to increased unemployment amongst graduates. Air emissions contribute slightly negatively to eco-technical change. This negative contribution is almost completely compensated by a slightly positive one to

[9] The materials balance principle highlights the crucial role of material inputs in generating pollution in production processes. It says that emissions are an inherent part of production activities using material resources. Furthermore, it says that materials can neither be created nor destroyed, but only change their form.

eco-efficiency change caused by a general reduction of air emissions resulting in an almost zero contribution to eco-productivity.

Finally, we point to avenues for future research. Firstly, with regard to the empirical part of the paper the model could be extended by additional types of pollutions and further kinds of abatement activities - depending on data availability - in order to complete the evaluation of environmental policy measures. Secondly, the proposed model could be a starting point to build a comprehensive model for analysing resource- or energy-efficiency and -productivity as a useful tool for investigating the possible impacts of the Europe 2020 goals (European Commission, 2010).

6 References

6.1 Literature

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6.3 Legislative texts

ARG, Arbeitsruhegesetz (Rest From Work Act of Austria).

AZG, Arbeitszeitgesetz (Working Time Act of Austria).

UrlG, Urlaubsgesetz (Holiday Act of Austria).

7 Appendix

The appendix shows the details of the derivation of the contributions of individual inputs and commodities to *EFFCH*, *TECHCH* and *PRODCH*.

1. Eco-efficiency change (*EFFCH*):

$$
\begin{aligned}
&EFFCH\left(z_t^0, y_{1,t}^0, y_{2,t}^0; z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) = \rho_t\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right) - \rho_{t+1}\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) = \\
&= \left(\sum_{j=1}^{n} u_{1,j;t,t} y_{1,j;t}^0 + \sum_{k=1}^{o} u_{2,k;t,t} y_{2,k;t}^0 + \sum_{i=1}^{m} v_{i;t,t} z_{i;t}^0\right) - \left(\sum_{j=1}^{n} u_{1,j;t+1,t+1} y_{1,j;t+1}^0 + \sum_{k=1}^{o} u_{2,k;t+1,t+1} y_{2,k;t+1}^0 + \sum_{i=1}^{m} v_{i;t+1,t+1} z_{i;t+1}^0\right) = \\
&= \sum_{j=1}^{n} u_{1,j;t,t} y_{1,j;t}^0 - \sum_{j=1}^{n} u_{1,j;t+1,t+1} y_{1,j;t+1}^0 + \sum_{k=1}^{o} u_{2,k;t,t} y_{2,k;t}^0 - \sum_{k=1}^{o} u_{2,k;t+1,t+1} y_{2,k;t+1}^0 + \sum_{i=1}^{m} v_{i;t,t} z_{i;t}^0 - \sum_{i=1}^{m} v_{i;t+1,t+1} z_{i;t+1}^0
\end{aligned}
$$

(28)

2. Eco-technical change (*TECHCH*):

$$
\begin{aligned}
&TECHCH\left(z_t^0, y_{1,t}^0, y_{2,t}^0, z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) = \\
&= \frac{1}{2}\left[\left(\rho_{t+1}\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) - \rho_t\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right)\right) + \left(\rho_{t+1}\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right) - \rho_t\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right)\right)\right] = \\
&= \frac{1}{2}\left[\left(\left(\sum_{j=1}^{n} u_{1,j;t+1,t+1} y_{1,j;t+1}^0 + \sum_{k=1}^{o} u_{2,k;t+1,t+1} y_{2,k;t+1}^0 + \sum_{i=1}^{m} v_{i;t+1,t+1} z_{i;t+1}^0\right) - \left(\sum_{j=1}^{n} u_{1,j;t,t+1} y_{1,j;t+1}^0 + \sum_{k=1}^{o} u_{1,k;t,t+1} y_{1,k;t+1}^0 + \sum_{i=1}^{m} v_{i;t,t+1} z_{i;t+1}^0\right)\right) + \right. \\
&\left. + \left(\left(\sum_{j=1}^{n} u_{1,j;t+1,t} y_{1,j;t}^0 + \sum_{k=1}^{o} u_{2,k;t+1,t} y_{2,k;t}^0 + \sum_{i=1}^{m} v_{i;t+1,t} z_{i;t}^0\right) - \left(\sum_{j=1}^{n} u_{1,j;t,t} y_{1,j;t}^0 + \sum_{k=1}^{o} u_{2,k;t,t} y_{2,k;t}^0 + \sum_{i=1}^{m} v_{i;t,t} z_{i;t}^0\right)\right)\right] = \\
&= \frac{1}{2}\left[\sum_{j=1}^{n} u_{1,j;t+1,t+1} y_{1,j;t+1}^0 - \sum_{j=1}^{n} u_{1,j;t,t+1} y_{1,j;t+1}^0 + \sum_{j=1}^{n} u_{1,j;t+1,t} y_{1,j,t}^0 - \sum_{j=1}^{n} u_{1,j;t,t} y_{1,j;t}^0 + \right. \\
&+ \sum_{k=1}^{o} u_{2,k;t+1,t+1} y_{2,k;t+1}^0 - \sum_{k=1}^{o} u_{2,k;t,t+1} y_{2,k;t+1}^0 + \sum_{k=1}^{o} u_{2,k;t+1,t} y_{2,k,t}^0 - \sum_{k=1}^{o} u_{2,k;t,t} y_{2,k;t}^0 + \\
&\left. + \sum_{i=1}^{m} v_{i;t+1,t+1} z_{i;t+1}^0 - \sum_{i=1}^{m} v_{i;t,t+1} z_{i;t+1}^0 + \sum_{i=1}^{m} v_{i;t+1,t} z_{i;t}^0 - \sum_{i=1}^{m} v_{i;t,t} z_{i;t}^0\right]
\end{aligned}
$$

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3. Eco-productivity change (*PRODCH*):

$$PRODCH\left(z_t^0, y_{1,t}^0, y_{2,t}^0, z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) =$$

$$\frac{1}{2}\left[\left(\rho_{t+1}\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right) - \rho_{t+1}\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right)\right) + \left(\rho_t\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right) - \rho_t\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right)\right)\right] =$$

$$= \frac{1}{2}\left[\left(\begin{array}{l}\left(\sum_{j=1}^{n} u_{1,j;t+1,t} y_{1,j;t}^0 + \sum_{k=1}^{o} u_{2,k;t+1,t} y_{2,k;t}^0 + \sum_{i=1}^{m} v_{i;t+1,t} z_{i;t}^0\right) - \\ -\left(\sum_{j=1}^{n} u_{1,j;t+1,t+1} y_{1,j;t+1}^0 + \sum_{k=1}^{o} u_{2,k;t+1,t+1} y_{2,k;t+1}^0 + \sum_{i=1}^{m} v_{i;t+1,t+1} z_{i;t+1}^0\right)\end{array}\right) + \right.$$

$$\left. + \left(\begin{array}{l}\left(\sum_{j=1}^{n} u_{1,j;t,t} y_{1,j;t}^0 + \sum_{k=1}^{o} u_{2,k;t,t} y_{2,k;t}^0 + \sum_{i=1}^{m} v_{i;t,t} z_{i;t}^0\right) - \\ -\left(\sum_{j=1}^{n} u_{1,j;t,t+1} y_{1,j;t+1}^0 + \sum_{k=1}^{o} u_{2,k;t,t+1} y_{2,k;t+1}^0 + \sum_{i=1}^{m} v_{i;t,t+1} z_{i;t+1}^0\right)\end{array}\right)\right] =$$

$$= \frac{1}{2}\left[\sum_{j=1}^{n} u_{1,j;t+1,t} y_{1,j;t}^0 - \sum_{j=1}^{n} u_{1,j;t+1,t+1} y_{1,j;t+1}^0 + \sum_{j=1}^{n} u_{1,j;t,t} y_{1,j;t}^0 - \sum_{j=1}^{n} u_{1,j;t,t+1} y_{1,j;t+1}^0 + \right.$$

$$+ \sum_{k=1}^{o} u_{2,k;t+1,t} y_{2,k;t}^0 - \sum_{k=1}^{o} u_{2,k;t+1,t+1} y_{2,k;t+1}^0 + \sum_{k=1}^{o} u_{2,k;t,t} y_{1,k;t}^0 - \sum_{k=1}^{o} u_{2,k;t,t+1} y_{2,k;t+1}^0$$

$$\left. + \sum_{i=1}^{m} v_{i;t+1,t} z_{i;t}^0 - \sum_{i=1}^{m} v_{i;t+1,t+1} z_{i;t+1}^0 + \sum_{i=1}^{m} v_{i;t,t} z_{i;t}^0 - \sum_{i=1}^{m} v_{i;t,t+1} z_{i;t+1}^0\right]$$

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