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Author: Mikulas Luptacik - Bernhard Mahlberg

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Efficiency change over time in a multisectoral economic system[1]

Mikulas Luptacik
Institute for Industrial Research, Vienna

and

Institute for Economic Policy,
University of Economics, Bratislava

Bernhard Mahlberg[#]
Institute for Industrial Research, Vienna

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Abstract

Neoclassical growth accounting is a methodology used to measure the contribution of different production factors to economic growth and to indirectly compute the rate of technological progress. This model assumes constant returns to scale and perfectly competitive factor markets, which implies that factor prices are equal to marginal products – something that is only satisfied if factor markets are cleared, and external effects and distortions are absent. However, these conditions are usually not satisfied in real economies. Moreover, growth accounting assumes efficiency on factor and commodity markets, and consequently does not distinguish between efficiency change and technical change. In this paper, we estimate total factor productivity growth without recourse to data on factor input shares or prices. In the proposed model, the economy is represented by the Leontief input-output model, which is extended by the constraints of primary inputs. A Luenberger productivity indicator is proposed to estimate productivity change over time; this is then decomposed in a way that enables us to examine the contributions of individual production factors and individual outputs to productivity change. The results allow the inference of which inputs or outputs of an economy are the drivers of the overall productivity change– this is then decomposed into efficiency change and technical change components. Using input-output tables of the US economy for the period 1977 to 2006, we show that technical progress is the main source of productivity change. Technical progress, in turn, is mostly driven by capital whereas low-skilled labor contributes negatively.

Keywords: Neoclassical Growth Accounting, Multi-Objective Optimization, Productivity Change, Efficiency Change, Luenberger Indicator
JEL codes: O47, C43, C61, C67

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[#] Corresponding author: Mahlberg@iwi.ac.at

1 Introduction

In the literature two approaches of productivity analysis can be found, namely the neoclassical approach and the frontier approach. Essentially, in both branches of the literature productivity is seen as the output-input ratio, whilst productivity growth is the residual between output growth and input growth. Under the neoclassical approach we refer to the seminal paper *Technical change and the aggregate production function* by Solow (1957). Neoclassical growth accounting aggregates growth rates of individual inputs by their respective partial production elasticities, which are quantified by their respective shares in total factor remuneration, thereby invoking the assumptions of perfectly competitive factor markets and constant returns to scale. This procedure requires data on factor input shares or prices. To obtain the evidence of the factor shares and the partial production elasticities, strong equilibrium requirements have to be satisfied. Factor prices must be equal to marginal products, an assumption which is only satisfied if factor markets are competitive, cleared and external effects as well as distortions originating from, for example, taxation are absent. Neoclassical growth accounting does not distinguish between efficiency change and technical change, and is not able to model multiple input/multiple output production processes.

The frontier approach can be implemented by different techniques. Within these techniques mathematical programming approaches and econometric approaches can be distinguished. The mathematical programming approaches are known as Data Envelopment Analysis (DEA) and Free Disposal Hall (FDH). Econometric approaches are referred to as Stochastic Frontier Approach (SFA). DEA is suitable for a multiple input/multiple output production, whereas the SFA is restricted to single output production. In macroeconomic productivity analysis, DEA is the most commonly used frontier approach. In contrast to growth accounting, DEA needs no factor price information and no equilibrium assumption necessary to equate price and marginal product. The weights required for aggregation of inputs and outputs are obtained as an integral part of the optimization process.

Although each approach tracks changes in the output-input ratio of an economy, the constructions are quite distinct. The neoclassical approach imputes productivity growth to factors, but cannot distinguish a movement towards the frontier and a movement of the frontier. The frontier approach allows decomposing productivity growth into a movement of the economy towards the efficiency frontier and a shift of the frontier. Productivity change is equal to efficiency change plus technical change. The frontier approach, however, is not capable of imputing value to factor inputs. In the paper by ten Raa and Mohnen (2002) a synthesis of both approaches is provided. They estimated total factor productivity (TFP) growth without recourse to data on factor input prices. In their work they reproduced the neoclassical TFP growth formulas, but within a framework in the spirit of Data Envelopment Analysis (DEA).

Another contribution to this topic is provided by Luptacik and Böhm (2010). Like ten Raa (1995, 2005) the economy is represented by the Leontief input-output model extended by the constraints for primary inputs. Using the multi-objective optimization model the efficiency frontier of the economy is generated. The solutions of the multi-objective optimization problems define efficient virtual decision making units (DMUs). The efficiency of the economy can be obtained as a solution of a DEA model with the virtual DMUs defining the potential and a DMU describing the actual performance of the economy. It can be proved that the solution of the above defined DEA model yields the same efficiency score and the same shadow prices as the models by ten Raa, despite the different variables used in both models. In this way the merits of both approaches can be utilized.

In this paper, the approach by Luptacik and Böhm (2010) is extended in two directions. Firstly, in the spirit of the Luenberger productivity indicator, the model is modified to an intertemporal approach providing the possibility to decompose total factor productivity change into efficiency change (catching up) and technology change (frontier shift). Secondly, productivity growth is decompounded in order to estimate the contribution of each individual primary input (labor, capital, etc.), as well as of each individual commodity. For illustration purposes, USA data for the observation period 1977 to 2006 are used and the following research questions are addressed: Is TFP change caused mainly by efficiency change or by technical change? What are the contributions of single factors to TFP change in the US economy?

The new approach allows computing productivity change for a single country. Before this, a balanced panel of quantity data for inputs and outputs of different countries were needed. Unlike Färe, et al. (1994), Henderson and Russell (2005) and Badunenko et al. (2008), our model does not determine an economy's frontier by benchmarking the production of a macro-aggregate on other economies: something which is problematic, when countries with different economic structures, development status, etc, are compared.

Our paper is structured as follows. Section 2 presents in detail the (static) model of Luptacik and Böhm (2010) and extends this model in line with the directional distance function approach. Section 3 presents our method to measure efficiency and productivity change over time; whilst Section 4 deals with an illustrative empirical application of the proposed model, with Section 5 left for our concluding remarks.

2 Leontief input-output model

2.1 The production possibility set

Leontief's input–output model conveniently describes the production relations of an economy in period t for a given nonnegative vector of final demand for n goods (y_t) produced in n interrelated sectors; gross output in period t of the sectors is denoted by the n-dimensional vector x_t. Production technology in period t is given by a indecomposable ($n \times n$) input coefficient matrix A_t. This in turn informs the use of a particular good i required for the production of a unit of good j, together with primary input requirements per unit of output in period t as given by ($m \times n$) matrix B_t. The use of primary inputs is restricted by the m-vector of available input quantities z_t in period t.

$$\begin{aligned}(I - A_t)x_t &\geq y_t \\ B_t x_t &\leq z_t\end{aligned} \tag{1}$$

In order to model multi-output/multi-input technologies, the notion of input and output distance functions can be used. For a single output this corresponds to the concept of a production function. Distance functions are well suited to define input and output oriented measures of technical efficiency. To work out such efficiency measures and to derive the output potential of an economy with n outputs we face in principle a multi objective optimization problem. In many cases such problems are reduced to a single objective optimization problem by suitable aggregation. For example, ten Raa (1995, 2005) uses world market prices for the n commodities employed in his model to reduce the optimization of n outputs to that of a single sum of values of the net products.

Pursuing the multiple objective approach Luptacik and Böhm (2010) propose to solve the following optimization model where each net output y_t is maximized, all be it subject to restraints on the availability of inputs z_t^0:

$$
\begin{aligned}
&\underset{x}{\text{Max}}\ y_t \\
&\text{s.t.} \\
&(I - A_t)x_t - y_t \geq 0 \\
&\qquad B_t x_t \qquad \leq z_t^0 \\
&x_t, y_t \geq 0
\end{aligned}
\tag{2}
$$

Luptacik and Böhm (2010) use the notation "Max" for a vector optimization problem and "max" for a single objective problem. They thus solve *n* single objective problems where final demand for each commodity is maximized, i.e.

$$
\max\ y_{t,j} \quad (j = 1,\ldots,n)
\tag{3}
$$

subject to the constraints in (2). For each of the *n* solutions of (3) denote the (also *n*-dimensional) solution vector x_t^{*j} (j = 1,...,n) representing the gross productions of commodities. The respective net-output column vectors are denoted y_t^{*j}.

Alternatively, for a given level of final demand y_t^0 the use of inputs z_t is minimized:

$$
\begin{aligned}
&\underset{x}{\text{Min}}\ z_t \\
&\text{s.t.} \\
&(I - A_t)x_t \qquad \geq y_t^0 \\
&\qquad B_t x_t - z_t^0 \leq 0 \\
&x_t, z_t \geq 0
\end{aligned}
\tag{4}
$$

In this case, therefore, *m* single objective problems are solved

$$
\min\ z_{t,i} \quad (i = 1,\ldots,m)
\tag{5}
$$

subject to the constraints in (4). The *m* solution vectors x_t^{*i} (i = 1,...,m) describe the gross production values of commodities for given final demand y_t^0 under the individual minimization of the primary inputs i = 1,...,m. The optimal input vectors are denoted by z_t^{*i}.

In the approach taken by Luptacik and Böhm (2010) these sets of values of both problems defined above are arranged column-wise in a pay-off matrix with the optimal values appearing in the main diagonal while the off-diagonal elements provide the levels of other sector net-outputs and inputs compatible with the individually optimized ones. The payoff matrix of dimension (n + m × n + m) is written

$$P_{t,t} = \begin{bmatrix} y_t^{*1} & y_t^{*2} & \ldots & y_t^{*n} & y_t^0 + s_y^1 \ldots y_t^0 + s_y^m \\ z_t^0 - s_z^1 & z_t^0 - s_z^2 & \ldots z_t^0 - s_z^n & & z_t^{*1} \quad \ldots \quad z_t^{*m} \end{bmatrix} \equiv \begin{bmatrix} P_{1;t,t} \\ P_{2;t,t} \end{bmatrix}$$

where s_y is the vector of the slack variables of the *n* outputs, s_z is the vector of the *m* input slacks and the subscript *t,t* indicates that in (3) and (5) the production technology as well as available inputs and final demand are observed in period *t*. Thus, each column of the payoff matrix containing either the maximal net output of a particular commodity for given inputs (the first *n* columns), or the minimal input for given final demand (the last *m* columns) yields an efficient solution (in the sense of Pareto-Koopmans). In this way the efficiency frontier of the economic system can be generated. In other words, the matrix $P_{t,t}$ relates the combinations of output quantities that are possible to produce for any given combination of inputs. In this way the "macroeconomic production function" for multi-input multi/output technologies can be described.

As shown by Belton and Vickers (1992, 1993) considering inputs and outputs as associated objectives by minimizing inputs and/or maximizing outputs the approaches of multiple criteria decision-making and Data Envelopment Analysis coincide – even though their ultimate aims may still differ).

Each of the points in the payoff-matrix $P_{t,t}$ is constructed independently of the other points, but take account of the entire systems relations. Knowing the efficient frontier the efficiency of the actual economy can be estimated. Each of the columns of the pay-off matrix can be seen as a virtual decision making unit with different input and output characteristics which are using the same production technique. The real economy as given by actual output and input data defines a new decision making unit whose distance to the frontier can be estimated.

For this purpose Luptacik and Böhm (2010) formulate the following input-oriented DEA model, measuring the efficiency of the economy described by the actual output and input data (y_t^0, z_t^0)

$$\begin{aligned} & \min_{\mu} \theta \\ & \text{s.t.} \\ & P_{1;t,t}\mu \geq y_t^0 \\ & -P_{2;t,t}\mu + \theta z_t^0 \geq 0 \\ & \mu \geq 0, \theta \geq 0 \end{aligned} \tag{6}$$

where $P_{1;t,t}$ is the output matrix and $P_{2;t,t}$ the input matrix. The columns of the matrix $P_{t,t}$ are the virtual DMUs, which represent the points of the efficient frontier. DMU_0 described by the actual output and input data (y_t^0, z_t^0) is not included in the description of the production possibility set, this is because the efficient points (the virtual DMUs) that enter into the evaluation are unaffected by such a removal. This is also true for an efficient DMU_0 that is on a part of the efficient frontier, but not an extreme point.

The question now arises as to how this approach is related to the neoclassical one of ten Raa (1995, 2005) and Debreu (1951).

2.2 The relationship between the DEA model and the LP-Leontief model

In the spirit of ten Raa (1995, 2005) and Debreu (1951), Luptacik and Böhm (2010) formulate the Leontief-model (1) as an optimization problem in the following way: minimize the use of primary inputs for given levels of final demand.

$$
\begin{aligned}
&\min_{x} \gamma \\
&\text{s.t.} \\
&(I - A_t)x_t \geq y_t^0 \\
&\quad - B_t x_t + \gamma z_t^0 \geq 0 \\
&x_t \geq 0,\ \gamma \geq 0
\end{aligned}
\tag{7}
$$

The parameter γ provides a radial efficiency measure. It records the degree by which primary inputs could be proportionally reduced but still capable of producing the required net outputs.

We rephrase both models of Luptacik and Böhm (2010) to models of non-oriented proportional directional distance function of technical efficiency in the following way:

$$
\begin{aligned}
\rho_t(z_t^0, y_t^0) = &\max_{\mu,\beta} \beta \\
&\text{s.t.} \\
&\quad -\beta y_t^0 + P_{1;t,t}\mu \geq y_t^0 \\
&\quad \beta z_t^0 + P_{2;t,t}\mu \leq z_t^0 \\
&\mu \geq 0,\ \beta \text{ free}
\end{aligned}
\tag{8}
$$

and

$$
\begin{aligned}
\omega_t(z_t^0, y_t^0) = &\max_{x,\delta} \delta \\
&\text{s.t.} \\
&\quad -\delta y_t^0 + (I - A_t)x_t \geq y_t^0 \\
&\quad \delta z_t^0 + B_t x_t \leq z_t^0 \\
&x_t \geq 0,\ \delta \text{ free}
\end{aligned}
\tag{9}
$$

The models (8) and (9) are based on the directional distance function which was proposed by Chambers et al. (1996b). In models (8) and (9) we consider a special case where we assume constant returns to scale (CRS) and $g^y = y_t^0$ as well as $g^z = z_t^0$, which yields us a non-oriented proportional measure of technical inefficiency. This is a radial measure which considers the proportional reductions in primary inputs and proportional extension of net output simultaneously.[2] The objective functions of models (8) and (9), i.e., ρ_t and ϖ_t represent the inefficiency scores of an economy. For an efficient economy $\rho_t = 0$ and $\varpi_t = 0$ and for an inefficient economy $\rho_t > 0$ and $\varpi_t > 0$.

[2] Model (8) and model (9) can be formulated as input-oriented by equating g^y with 0 or as output-oriented model by equating g^z with 0. For input and output oriented models all statements of the rest of this section pertain analogously.

Taking into account the interpretation of the inefficiency parameters β in the DEA model (8) and δ in the Leontief model (9) it can be seen that despite the different model formulations the objective functions are similar. Both models measure the efficiency of the economy by radial reduction of primary inputs, as well as radial expansion of net output for given amounts of resources and final demand in the economy. The relationships between (8) and (9) are given by the following proposition.

Proposition 1: *The efficiency score* ρ_t *of DEA problem (8) is exactly equal to the efficiency measure* ϖ_t *of LP-model (9). The dual solution of model (9) coincides with the solution of the DEA multiplier problem which is the dual of problem (8).*

Proof The dual model to (9) can be written

$$
\begin{aligned}
&\min \; p'_{t,t}\,\mathrm{y}_t^0 + r'_{t,t}\,z_t^0 \\
&\text{s.t.} \\
&\quad p'_{t,t}(I-A_t) + r'_{t,t}B_t \geq 0 \\
&\quad -p'_{t,t}\,y_t^0 + r'_{t,t}\,z_t^0 = 1 \\
&\quad p_{t,t} \leq 0,\; r_{t,t} \geq 0
\end{aligned}
\tag{10}
$$

where $p_{t,t}$ are the shadow prices of the n commodities, and r the shadow prices of the m primary inputs. From indecomposability of A_t follows for the Leontief model that, for $y_t^0 \geq 0$, $x_t > 0$ and $(I-A_t)^{-1} > 0$ (cf. e.g. Nikaido 1968). From the complementary slackness theorem follows

$$p'_{t,t}(I-A_t) + r'_{t,t}B_t = 0$$

and thus

$$p'_{t,t} = -r'_{t,t}B_t(I-A_t)^{-1} \leq 0$$

which has a clear economic interpretation. Matrix $B_t(I-A_t)^{-1}$ contains the cumulative requirements of primary inputs. Therefore the total values of used primary inputs determine the shadow prices of commodities $p_{t,t}$ but because (10) minimizes inefficiency, these shadow prices are non-positive. The Shadow prices $r_{t,t}$ describe the effect of a change in primary inputs on inefficiency and are non-negative. So in other words, a ceteris paribus increase of final demand by a small amount may reduce inefficiency; while a ceteris paribus increase of primary input, by a similarly small amount, may raise inefficiency.

The dual to (8) is

$$
\begin{aligned}
&\min \; u'_{t,t}\,\mathrm{y}_t^0 + v'_{t,t}\,z_t^0 \\
&\text{s.t.} \\
&\quad u'_{t,t}P_{1;t,t} + v'_{t,t}P_{2;t,t} \geq 0 \\
&\quad -u'_{t,t}\,y_t^0 + v'_{t,t}\,z_t^0 = 1 \\
&\quad u_{t,t} \leq 0,\; v_{t,t} \geq 0
\end{aligned}
\tag{11}
$$

Again, the different signs of shadow prices $u_{t,t}$ and $v_{t,t}$ correspond to different impacts of increasing primary inputs and increasing final demand.

Multiplying the Leontief inverse by the matrix of generated net outputs P_1 we obtain the corresponding total gross output requirements, denoted by matrix T :

$$(I - A_t)^{-1} P_{1;t,t} = T > 0 \tag{12}$$

In other words T represents the total output requirements for each virtual decision making unit. Consequently

$$B_t T = B_t (I - A_t)^{-1} P_{1;t,t}$$

gives the necessary amount of primary inputs to satisfy the generated total output requirements. This coincides with the construction of matrix $P_{2;t,t}$ describing the total primary input requirements necessary to satisfy final demands $P_{1;t,t}$. Therefore

$$P_{2;t,t} = B_t T \tag{13}$$

It follows from (12) that

$$P_{1;t,t} = (I - A_t) T \tag{14}$$

Multiplying the first constraint in (10) by T yields

$$p'_{t,t} (I - A_t) T + r'_{t,t} B_t T \geq 0 \tag{15}$$

Substituting (13) and (14) for $P_{2;t,t}$ and $P_{1;t,t}$ respectively into (15) we obtain exactly the constraints as of the dual problem (11):

$$p'_{t,t} P_{1;t,t} + r'_{t,t} P_{2;t,t} \geq 0$$

Since we have two linear programming problems, which both have the same constraints and the same coefficients in the objective functions, the optimal values of the objective functions must also be the same: $-p'_{t,t} y_t^0 + r'_{t,t} z_t^0 = -u'_{t,t} y_t^0 + v'_{t,t} z_t^0$. Consequently $p'_{t,t} = u'_{t,t}$ as well as $r_{i;t,t} = v_{i;t,t}$ and according to the duality theorem of linear programming $\varpi_t(z_t^0, y_t^0) = \rho_t(z_t^0, y_t^0)$.

3 Efficiency change of the economy over time

Following Chambers et al. (1996a,b), the non-oriented proportional Luenberger indicator is defined over two accounting periods (*t* and *t*+1) as:

$$L\left(z_t^0, y_t^0, z_{t+1}^0, y_{t+1}^0\right) = \frac{1}{2}\left[\left(\rho_{t+1}\left(z_t^0, y_t^0\right) - \rho_{t+1}\left(z_{t+1}^0, y_{t+1}^0\right)\right) + \left(\rho_t\left(z_t^0, y_t^0\right) - \rho_t\left(z_{t+1}^0, y_{t+1}^0\right)\right)\right] \quad (16)$$

The non-oriented proportional Luenberger indicator can be decomposed into efficiency change (catch-up, EFFCH) and technical change (frontier shift, TECHCH) as follows:

$$EFFCH\left(z_t^0, y_t^0, z_{t+1}^0, y_{t+1}^0\right) = \rho_t\left(z_t^0, y_t^0\right) - \rho_{t+1}\left(z_{t+1}^0, y_{t+1}^0\right) \quad (17)$$

$$TECHCH\left(z_t^0, y_t^0, z_{t+1}^0, y_{t+1}^0\right) = \frac{1}{2}\left[\left(\rho_{t+1}\left(z_{t+1}^0, y_{t+1}^0\right) - \rho_t\left(z_{t+1}^0, y_{t+1}^0\right)\right) + \left(\rho_{t+1}\left(z_t^0, y_t^0\right) - \rho_t\left(z_t^0, y_t^0\right)\right)\right] \quad (18)$$

This Luenberger indicator is expressed as the sum of EFFCH and TECHCH. EFFCH captures the average gain/loss in inputs and net outputs due to a difference in technical efficiency from period t to period t+1. TECHCH captures the average gain/loss in inputs and net outputs due to a shift in technology from period t to period t+1.

To compute the non-oriented proportional Luenberger indicator and its components, besides the estimation of two own-period inefficiency scores, i.e. $\rho_t\left(z_t^0, y_t^0\right)$ and $\rho_{t+1}\left(x_{t+1}^0, y_{t+1}^0\right)$, we need the estimation of two cross-period inefficiency scores:

1) $\rho_{t+1}\left(z_t^0, y_t^0\right)$, which represents the degree of inefficiency of an economy operating at t when evaluated with respect to the technology at t+1; and

2) $\rho_t\left(z_{t+1}^0, y_{t+1}^0\right)$, which represents the degree of inefficiency of an economy at t+1 when evaluated with respect to the technology at t.

First, the linear programming (LP) program in (8) is solved for two periods (t and t+1) to arrive at $\rho_t\left(z_t^0, y_t^0\right)$ and $\rho_{t+1}\left(z_{t+1}^0, y_{t+1}^0\right)$. For these LPs, separate output matrices P_1, i.e. $P_{1;t,t}$ and $P_{1;t+1,t+1}$, and separate input matrices P_2, i.e. $P_{2;t,t}$ and $P_{2;t+1,t+1}$, have to be constructed by solving the LPs (3) and (5) for each period.

Second, the cross-period distance function, $\rho_{t+1}\left(z_t^0, y_t^0\right)$ can be set up as

$$\begin{aligned} \rho_{t+1}\left(z_t^0, y_t^0\right) &= \max_{\mu,\beta} \beta \\ &\text{s.t.} \\ &\quad -\beta y_t^0 + P_{1;t+1,t}\mu \geq y_t^0 \\ &\quad \beta z_t^0 + P_{2;t+1,t}\mu \leq z_t^0 \\ &\quad \mu \geq 0,\ \beta \text{ free} \end{aligned} \quad (19)$$

Similarly, the other cross-period distance function, $\rho_t\left(x_{t+1}^0, y_{t+1}^0\right)$ can be set up as

$$\begin{aligned} \rho_t\left(z_{t+1}^0, y_{t+1}^0\right) &= \max_{\mu,\beta} \beta \\ &\text{s.t.} \\ &\quad -\beta y_{t+1}^0 + P_{1;t,t+1}\mu \geq y_{t+1}^0 \end{aligned} \quad (20)$$

$$\beta z_{t+1}^0 + P_{2;t,t+1}\text{“} \le z_{t+1}^0$$
$$\mu \ge 0,\ \beta \text{ free}$$

For these two DEA models separate output matrices P_1, i.e. $P_{1;t+1,t}$ and $P_{1;t,t+1}$, and separate input matrix P_2, i.e. $P_{2;t+1,t}$ and $P_{2;t,t+1}$, have to be constructed by solving the LPs (3) and (5). For these computations, production technology on the one hand and primary input endowment as well as final demand on the other are observed in different periods, which is indicated by the subscripts t+1,t and t,t+1. In total, the LPs (3) and (5) have to be solved four times.

In the case of the cross-period LP programs, $\rho_{t+1}\left(z_t^0, y_t^0\right)$ in (19) and $\rho_t\left(z_{t+1}^0, y_{t+1}^0\right)$ in (20), when the economy under evaluation remains outside the technology set it is considered ‘super efficient’, meaning the inefficiency score β becomes negative. Such an inefficiency score implies that the primary inputs need to be increased and net outputs need to be decreased to get such super efficient economies projected onto the efficient frontier.

The proposed method allows the researcher to examine the reasons of EFFCH, TECHCH and total factor productivity change (TFPCH). It attributes the use of individual primary input and individual commodity to productivity change and its components. To show this we start first by deriving the formula for EFFCH, before we present the formulas for TECHCH and TFPCH.

The starting points of this analysis are the definition of efficiency change and the dual to the DEA model (8) as it is shown in model (11).

$$\begin{aligned} &EFFCH\left(z_t^0, y_t^0, z_{t+1}^0, y_{t+1}^0\right) = \rho_t\left(z_t^0, y_t^0\right) - \rho_{t+1}\left(z_{t+1}^0, y_{t+1}^0\right) = \\ &= \left(\sum_{j=1}^n u_{j;t,t} y_{j;t}^0 + \sum_{i=1}^m v_{i;t,t} z_{i;t}^0\right) - \left(\sum_{j=1}^n u_{j;t+1,t+1} y_{j;t+1}^0 + \sum_{i=1}^m v_{i;t+1,t+1} z_{i;t+1}^0\right) = \\ &= \sum_{j=1}^n u_{j;t,t} y_{j;t}^0 - \sum_{j=1}^n u_{j;t+1,t+1} y_{j;t+1}^0 + \sum_{i=1}^m v_{i;t,t} z_{i;t}^0 - \sum_{i=1}^m v_{i;t+1,t+1} z_{i;t+1}^0 \end{aligned} \tag{21}$$

It turns out that the contribution of the i-th primary input is

$$v_{i;t,t} z_{i;t}^0 - v_{i;t+1,t+1} z_{i;t+1}^0 \tag{22}$$

and that of the j-th commodity

$$u_{j;t,t} y_{j;t}^0 - u_{j;t+1,t+1} y_{j;t+1}^0 \tag{23}$$

The sum of the contributions of all primary inputs and all commodities is exactly equal to EFFCH.

For TECHCH, our starting points are the definition of technological change and the duals to the DEA models (8), (19) and (20).

$$TECHCH\left(z_t^0,y_t^0,z_{t+1}^0,y_{t+1}^0\right)=\frac{1}{2}\left[\left(\rho_{t+1}\left(z_{t+1}^0,y_{t+1}^0\right)-\rho_t\left(z_{t+1}^0,y_{t+1}^0\right)\right)+\left(\rho_{t+1}\left(z_t^0,y_t^0\right)-\rho_t\left(z_t^0,y_t^0\right)\right)\right]=$$

$$=\frac{1}{2}\left[\left(\left(\sum_{j=1}^{n}u_{j;t+1,t+1}y_{j;t+1}^0+\sum_{i=1}^{m}v_{i;t+1,t+1}z_{i;t+1}^0\right)-\left(\sum_{j=1}^{n}u_{j;t,t+1}y_{j;t+1}^0+\sum_{i=1}^{m}v_{i;t,t+1}z_{i;t+1}^0\right)\right)+\right.$$

$$\left.+\left(\left(\sum_{j=1}^{n}u_{j;t+1,t}y_{j;t}^0+\sum_{i=1}^{m}v_{i;t+1,t}z_{i;t}^0\right)-\left(\sum_{j=1}^{n}u_{j;t,t}y_{j;t}^0+\sum_{i=1}^{m}v_{i;t,t}z_{i;t}^0\right)\right)\right]= \tag{24}$$

$$=\frac{1}{2}\left[\sum_{j=1}^{n}u_{j;t+1,t+1}y_{j;t+1}^0-\sum_{j=1}^{n}u_{j;t,t+1}y_{j;t+1}^0+\sum_{j=1}^{n}u_{j;t+1,t}y_{j,t}^0-\sum_{j=1}^{n}u_{j;t,t}y_{j;t}^0+\right.$$

$$\left.+\sum_{i=1}^{m}v_{i;t+1,t+1}z_{i;t+1}^0-\sum_{i=1}^{m}v_{i;t,t+1}z_{i;t+1}^0+\sum_{i=1}^{m}v_{i;t+1,t}z_{i;t}^0-\sum_{i=1}^{m}v_{i;t,t}z_{i;t}^0\right]$$

Therefore the contribution of the *i*-th primary input is given by

$$\frac{1}{2}\left(v_{i;t+1,t+1}z_{i;t+1}^0-v_{i;t,t+1}z_{i,t+1}^0+v_{i;t+1,t}z_{i;t}^0-v_{i;t,t}z_{i;t}^0\right) \tag{25}$$

and that of the *j*-th commodity

$$\frac{1}{2}\left(u_{j;t+1,t+1}y_{j;t+1}^0-u_{j;t,t+1}y_{j;t+1}^0+u_{j;t+1,t}y_{j;t}^0-u_{j;t,t}y_{j;t}^0\right) \tag{26}$$

The sum of the contributions of all primary inputs and all commodities is exactly equal to TECHCH.

For TFPCH we begin with the definition of total factor productivity change and the duals to the DEA models (8), (19) and (20).

$$L\left(z_t^0,y_t^0,z_{t+1}^0,y_{t+1}^0\right)=\frac{1}{2}\left[\left(\rho_{t+1}\left(z_t^0,y_t^0\right)-\rho_{t+1}\left(z_{t+1}^0,y_{t+1}^0\right)\right)+\left(\rho_t\left(z_t^0,y_t^0\right)-\rho_t\left(z_{t+1}^0,y_{t+1}^0\right)\right)\right]=$$

$$=\frac{1}{2}\left[\left(\left(\sum_{j=1}^{n}u_{j;t+1,t}y_{j;t}^0+\sum_{i=1}^{m}v_{i;t+1,t}z_{i;t}^0\right)-\left(\sum_{j=1}^{n}u_{j;t+1,t+1}y_{j;t+1}^0+\sum_{i=1}^{m}v_{i;t+1,t+1}z_{i;t+1}^0\right)\right)+\right.$$

$$\left.\left(\left(\sum_{j=1}^{n}u_{j;t,t}y_{j;t}^0+\sum_{i=1}^{m}v_{i;t,t}z_{i;t}^0\right)-\left(\sum_{j=1}^{n}u_{j;t,t+1}y_{j;t+1}^0+\sum_{i=1}^{m}v_{i;t,t+1}z_{i;t+1}^0\right)\right)\right]=$$

$$=\frac{1}{2}\left[\sum_{j=1}^{n}u_{j;t+1,t}y_{j;t}^0-\sum_{j=1}^{n}u_{j;t+1,t+1}y_{j;t+1}^0+\sum_{j=1}^{n}u_{j;t,t}y_{j;t}^0-\sum_{j=1}^{n}u_{j;t,t+1}y_{j;t+1}^0+\right.$$

$$\left.+\sum_{i=1}^{m}v_{i;t+1,t}z_{i;t}^0-\sum_{i=1}^{m}v_{i;t+1,t+1}z_{i;t+1}^0+\sum_{i=1}^{m}v_{i;t,t}z_{i;t}^0-\sum_{i=1}^{m}v_{i;t,t+1}z_{i;t+1}^0\right]$$

(27)

Hence, the contribution of the *i*-th primary input is given by

$$\frac{1}{2}\left(v_{i;t+1,t}z_{i;t}^{0}-v_{i,t+1,t+1}z_{i;t+1}^{0}+v_{i,t,t}z_{i;t}^{0}-v_{i,t,t+1}z_{i;t+1}^{0}\right) \tag{28}$$

and that of the j-th commodity by

$$\frac{1}{2}\left(u_{j;t+1,t}y_{j;t}^{0}-u_{j;t+1,t+1}y_{j;t+1}^{0}+u_{j;t,t}y_{j;t}^{0}-u_{j;t,t+1}y_{j;t+1}^{0}\right) \tag{29}$$

The sum of the contributions of all primary inputs and all commodities is exactly equal to EFFCH.

It can be shown that the contribution of the i-th primary input to EFFCH plus the contribution of the i-th primary input to TECHCH is equal to the contribution of the same primary input to TFPCH. Furthermore, the contribution of the j-th commodity to EFFCH plus the contribution of the j-th commodity to TECHCH is equal to the contribution of the same commodity to TFPCH.

This decomposition enables the researcher to empirically examine the contributions of each individual primary input and commodity towards the productivity change and its components—efficiency change and technical change.

4 An Illustrative Empirical Application

In this section, we describe how our techniques can be used to estimate the long-term total factor productivity growth in the United States of America (USA), in order to illustrate the applicability of our proposed approach. In order to investigate their meaning for productivity growth, we compute the contributions of different primary inputs and of individual commodities.

4.1 Data

Our data set comprises the two most important primary inputs: labor and capital. Labor is decomposed into high-skilled, medium-skilled and low-skilled, and classified according to the educational attainment level. Thus, high-skilled labor is defined as workers who graduated from college and above; medium-skilled as workers who graduated from high school and have some years of college, but not completed; and low-skilled, as workers who are less than high school educated or with some years of high school, but again not completed (cf. Timmer et al., 2007). Capital is represented by aggregate capital stock, and contains all asset types – including residential structures, non-residential structures, infrastructure, transportation equipment, computing equipment, communications equipment. Final demand serves as the output measure and consists of six aggregates of commodities, i.e. agriculture, construction, manufacturing, trade, transportation & utilities, services and others. Hence, our empirical model consists of four primary inputs: labor of three skill types, and capital. We consider these four inputs as the most important production factors. Since our model allows for the handling of multiple inputs, it would be possible to add other indicators like primary energy and natural resources.

The interrelationship between the industries is measured by input-output tables that are based on domestic use tables as well as make tables. Because our analyses are done mainly for illustration purposes we content ourselves with rather highly aggregated input-output tables. More detailed analyses would be possible since our model allows treating multiple outputs. From these input-output tables the input coefficient matrices (A-matrices) as well as the matrices of primary input requirements per unit of gross output (B-matrices), are computed (see Table 3). The observation period goes from 1977 to 2006.

Tab. 1: Descriptive statistics of primary inputs

	resources used (1)	endowment (2)	utilization (=(1)/(2))
	in 1977		
High-skilled labor (in Mill. hours)	32,021	38,353	0.83
Medium-skilled labor (in Mill. hours)	101,139	220,071	0.46
Low-skilled labor (in Mill. hours)	41,098	52,607	0.78
Capital, all assets (in Bill. USD)	12,949	15,530	0.83
	in 2006		
High-skilled labor (in Mill. hours)	80,086	100,773	0.79
Medium-skilled labor (in Mill. hours)	147,965	307,790	0.48
Low-skilled labor (in Mill. hours)	24,934	36,030	0.69
Capital, all assets (in Bill. USD)	29,278	36,429	0.80

Table 1 shows descriptive statistics of primary inputs. Labor is measured in millions of hours worked, and capital in billion of US$. Capital is represented by real fixed-capital stock at prices for the year 1995. For high and medium-skilled labor as well as capital the quantities used and the endowments clearly increased. For low-skilled labor, quantity used as well as endowment clearly decreased. Furthermore, from Table 1 it can be seen that the resource utilization (i.e. ratio of resources used to endowment) of low-skilled and high-skilled labor as well as of capital stock worsened, whereas of medium-skilled labor improved. From these data we see that the utilization of resources decreased by tendency and we expect an increase in inefficiency of the whole economy, and therefore efficiency regress indicated by Luenberger indicator.

Measured in billions of US$, Table 2 (below) presents the data on final demand. From the table it can be seen that the final demand for all commodities increased, and growth rate differs from commodity to commodity. The increases of demand for commodities from the tertiary sector are the highest, and from the primary sector the lowest. Consequently, the composition of final demand changed considerably.

Tab. 2: Descriptive statistics of final demand

	Final demand in 1977	Final demand in 2006	Final demand growth rate
	in bill. USD		in percent
Agriculture	44	48	9.23
Construction	756	1,220	61.40
Manufacturing	1,215	1,691	39.21
Trade, transportation & utilities	821	2,130	159.40
Services	2,046	6,054	195.93
Others	582	2,063	254.79
Total	**5,462**	**13,205**	**141.75**

The data source of labor used and capital used is the EU KLEMS Growth and Productivity Accounts Data base (March 2008 Release for labor and November 2009 Release for capital). The time series were downloaded in November 2011. These data serve as a basis for computation of the primary input requirement matrices (B-matrices) of the respective years. The labor endowment of the US economy cannot be observed directly. Therefore, we have to estimate them by applying the following procedure. In a first step, we take data on the population of different skill levels of the respective years from Lutz et al (2007) and Samir et al. (2010). To come close to the definition of working-age population from the US Bureau of Labor Statistics we take the entire population of all persons 15 years and older as labor endowment. These data are given in number of persons. In order to convert these data into potential labor input measured in number of hours, in a second step, we multiply them by number of hours usually worked per day (and workers)[3], and by the number of days usually worked per year (and worker)[4]. The capital endowment cannot be observed directly and has to be estimated as well. This estimation is done by multiplying the capital stock used taken from EUKLEMS by capacity utilization rate for total industry. Data on capacity utilization are taken from Federal Reserve (2012), while final demand data (together with the input-output tables) are taken from Miller and Blair (2009) and were deflated by applying the approach developed by Koller and Stehrer (2010).

Tab. 3: Primary input requirement matrices (B-matrices)

	Agri-culture	Constru-ction	Manufac-turing	Trade, Transport & Utilities	Services	Others	**mean**
	in 1977						
high-skilled labor	3.18	0.73	1.51	3.40	2.09	12.14	**3.26**
medium-skilled labor	20.86	5.61	7.65	18.18	5.44	21.77	**10.28**
low-skilled labor	19.12	3.22	4.30	6.04	1.47	7.02	**4.18**
capital total	1.95	0.11	0.37	1.24	2.34	1.79	**1.32**
	in 2006						
high-skilled labor	4.03	1.45	1.53	3.13	2.14	10.77	**3.24**
medium-skilled labor	14.72	8.76	3.78	10.07	2.93	13.82	**5.98**
low-skilled labor	5.79	2.73	0.75	1.55	0.49	1.42	**1.01**
capital total	1.27	0.13	0.33	0.82	1.58	1.96	**1.18**

Table 3 presents the primary input requirement matrices of 1977 and 2006. Input requirements are defined as the ratio of amount of primary inputs used in a sector, divided by cross output produced of a sector and tell how much of a resource is needed to produce one unit of output. It is exactly the reciprocal of the single factor productivity (e.g. labor productivity). A decrease over time indicates an increase of the productivity of this factor in the respective sector. In this case, fewer resources are required to produce one unit of output. From this table it can be seen that in most sectors, as well as on average, the values of primary input requirements decrease. Based on these values we expect that the Luenberger Indicator will indicate technical progress.

4.2 Results

First of all we compute the inefficiency scores and the shadow prices for the years 1977 and 2006 applying the DEA model [(8) and (11)] and the Leontief model [(9) and (10)] in order to show empirically that the results of both models are equal. As can be seen from Table 4, this is indeed the case, and the empirical results confirm the statement of proposition 1. Our results in Table 4 (first line below the column heading) show an inefficiency score of 0.090 and 0.109 in years 1977 and 2006, respectively. These results can be interpreted as follows: in both years the US economy is inefficient, in the sense that its actual performance deviates from its potential and its resources are not fully utilized. In 1977 the US economy could increase its actual final demand and decrease the actual use of primary inputs by round 9 percent simultaneously. In 2006, the US economy is even more inefficient, further away from its possibilities, and its potential for improvement was larger than in 1977. It could raise the actual final demand and reduce the primary inputs actually used by around 11 percent.

Additionally, Table 4 shows the results of shadow prices computations. According to the interpretation discussed in section 2.2, positive shadow prices of primary input indicate that an increase in the endowment raises inefficiency. Conversely, a negative shadow price of a commodity reveals that an increase in final demand reduces inefficiency. Generally speaking, a

[3] These are normally eight hours.
[4] These are normally 236 days (= 365 calendar days – 104 week end days – 15 vacation days – 10 holidays).

non-zero shadow price indicates that the respective resource or commodity is scarce. By contrast, a value of zero implies that a change in endowment or final demand does not change inefficiency and shows the respective resource or commodity is abundant. The results presented in Table 4 indicate clearly that in 1977 high-skilled labour is scarce, whereas the other primary inputs are abundant. An additional unit of high-skilled labour (with all other things held constant) raises inefficiency, whereas an increase of all other primary inputs does not change inefficiency. Although through the period 1977 to 2006 the picture obviously changes, as high-skilled labour becomes abundant and capital scarce. Furthermore, according to the shadow prices listed in Table 4 an increase in final demand for any commodity decreases inefficiency in both years indicating they are all of them are scarce. In 1977 the shadow prices are quite unequal. From 1977 to 2006 the shadow prices get more equal.

Tab. 4: Inefficiency scores and shadow prices from single period DEA and Leontief model for 1977 and 2006

	1977		**2006**	
	DEA-model	Leonief-model	DEA-model	Leonief-model
inefficiency scores	0.090	0.090	0.109	0.109
	shadow prices			
High-skilled labor	0.00001	0.00001	0	0
Medium-skilled labor	0	0	0	0
Low-skilled labor	0	0	0	0
capital	0	0	0.00002	0.00002
Agriculture	-0.00013	-0.00013	-0.00004	-0.00004
Construction	-0.00005	-0.00005	-0.00002	-0.00002
Manufacturing	-0.00011	-0.00011	-0.00003	-0.00003
Trade, transportation & utilities	-0.00009	-0.00009	-0.00003	-0.00003
Services	-0.00005	-0.00005	-0.00004	-0.00004
Other	-0.00018	-0.00018	-0.00004	-0.00004

Note: DEA model … models (8) and (11), Leontief model … models (9) and (10)

In a next step, we apply our DEA models and the Luenberger Indicator to estimate the total factor productivity change in the US economy from 1977 to 2006. The results are shown in Table 5. Again, we apply our DEA model as well as the Leontief model, and compare the results in order to check whether the outcomes coincide. The results of both models are exactly the same. The inefficiency scores of single period and of mixed period are equal. As a consequence, the values of EFFCH, TECHCH and TFPCH are the same too. According to our results the efficiency of the US economy slightly decreases. The efficiency change score is around minus 2 percent indicating efficiency regress. This result confirms our expectation we have drawn from Table 1. Contrary to efficiency regress we find a positive technical change score of around 27 percent. This value shows that the US economy goes through a very clear technical progress during the observation period. This result confirms our expectations we have drawn from Table 3. The sum of efficiency change and technical change is equal to TFPCH. The productivity progress of the US economy amounts to around 25 percent according to both models.

Tab. 5: Results of Luenberger Indicator and its components, 1977 to 2006

		DEA model	Leontief model
Inefficiency scores	in 1977	0.090	0.090
	in 2006	0.109	0.109
	1977 to 2006 (mixed period)	-0.418	-0.418
	2006 to 1977 (mixed period)	0.093	0.093
Luenberger Indicator	Efficiency change (EFFCH)	**-0.019**	**-0.019**
	Technical change (TECHCH)	**0.265**	**0.265**
	Total factor productivity change (TFPCH)	**0.246**	**0.246**

These results raise the question as to which primary input cause the EFFCH, TECHCH and TFPCH. Or in other words, what are the contributions of the individual inputs and outputs to efficiency, technology and productivity development. To answer these questions, we applied the approach described in the previous section, i.e. the formula (22), (23), (25), (26), (28), and (29), which combines shadow prices and observed data (endowment of primary input and final demand for commodities) to estimate the contribution of individual primary input. The results are shown in Table 6 and Figure 1. The sums of the columns are equal to the EFFCH score, TECHCH score and TFPCH score, respectively.

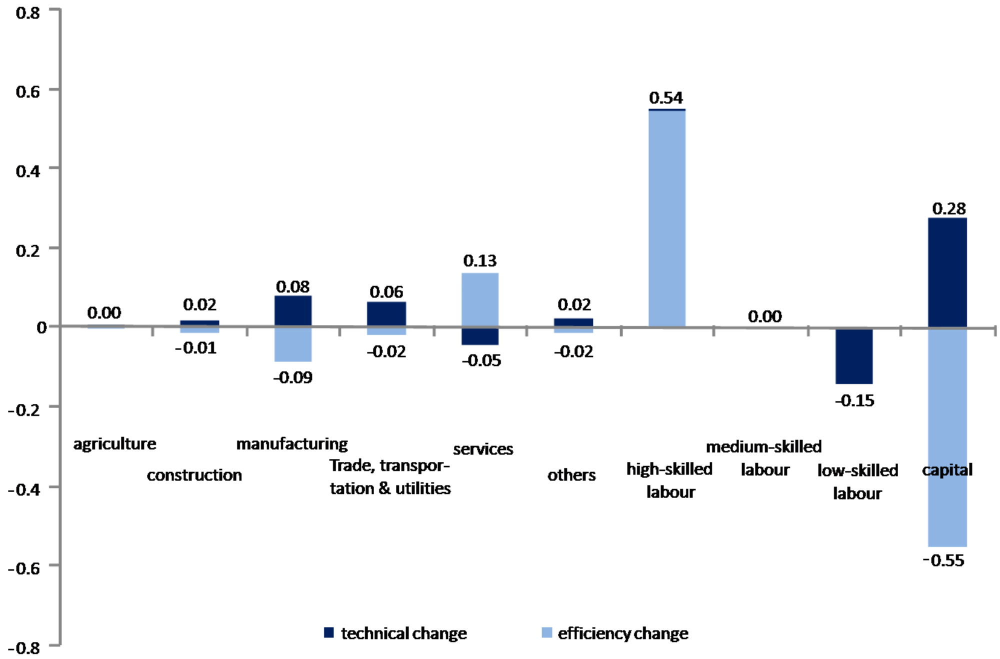
Tab. 6: contribution of each individual output and primary input

		Efficiency change (EFFCH)	Technical change (TECHCH)	Total factor Productivity change (TFPCH)
Final demand (net outputs)	agriculture	-0.004	0.006	0.002
	construction	-0.013	0.017	0.004
	manufacturing	-0.086	0.075	-0.010
	Trade, transportation & utilities	-0.023	0.062	0.039
	services	0.133	-0.048	0.086
	Other	-0.017	0.020	0.003
primary inputs (resources)	high-skilled labor	0.545	0.001	0.546
	medium-skilled labor	0	0	0
	low-skilled labor	0	-0.146	-0.146
	capital	-0.554	0.277	-0.277
		-0.019	**0.265**	**0.246**

It turns out that efficiency change is driven by decline in use of resources, as well as in final demand. Among the primary inputs high-skilled workers' contributions are clearly positive. Conversely, capital contributes negatively, and almost compensates for the positive contribution of high-skilled labor. This result hints to a substitution effect between capital and high-skilled labor. This relationship reflects the change in terms of shortage, which can be seen from the shadow prices in Table 4. In 1977 the shadow price of high-skilled labor is different from zero and of capital is equal to zero. In 2006 it is completely reverse. The contributions of the other two primary inputs are zero as both of them are abundant in each year and therefore the shadow prices are equal to zero. The contributions of five out of six commodities are slightly negative. Only service contributes in a clearly positive way, and overcompensates for the negative contribution of the other outputs. The increase of quantity of services obviously outweighs the decline of shadow prices. The decrease of shadow prices observed for all commodities (cf. Table 4) predominates the increase in demanded quantity of all commodities except services.

Technical regress is driven by primary inputs as well as outputs, the total contributions of both categories being positive. Among the primary inputs, capital contributes the most whereas high-skilled labor contributes almost nothing. Meanwhile, the contribution of low-skilled labor is negative, with the contribution of medium-skilled labor equating to exactly zero. The relationship between the contributions of low-skilled workers and capital reflects the well-known substitution between these factors. Medium-skilled labor does not contribute because its shadow prices are always zero, showing that it is never a scarce resource. The contributions of final demand for any commodity are marginally positive, with the exception of the slightly negative contribution of services.

Figure 1: Contribution of individual commodities and primary inputs to efficiency change and technical change



In Table 6 the rightmost column shows the contribution to TFPCH. The contribution to TFPCH is exactly the row sum, and thus, the sum of the contributions of individual factors to efficiency change and technical change. The total contributions of primary inputs and final demand to TFPCH are clearly positive. High-skilled labor contributes positively, while low-skilled labor along with capital negatively. The contributions of primary inputs are large relative to that of commodities, whose contributions are small but mostly positive – the highest of which being services.

5 Conclusions

Neoclassical growth accounting is a methodology used to measure the contribution of different factors to economic growth and to indirectly compute the rate of technological progress. This procedure, introduced by Solow (1957), breaks the growth rate of total output down into two constituent parts. Namely, that which is due to increases in the amount of production factors used (i.e. labor, capital, etc.), and that which cannot be accounted for by observable changes in factor usage. Often referred to as the Solow residual, this second and less apparent part is taken to represent increases in total factor productivity, or as an indicator of technological progress, and measured as a residuum between output growth and the weighted growth of production factors – these weighted factors represented by share of total income as to different production factors. This model assumes constant returns to scale and perfectly competitive factor markets, which implies that factor prices are equal to marginal products – something that is only satisfied if factor markets are cleared, and external effects and distortions are absent. However, these conditions are usually not satisfied in real economies. Moreover, growth accounting assumes efficiency on factor and commodity markets, and consequently does not distinguish between efficiency change and technical change.

As an alternative to neoclassical growth accounting a frontier approach is proposed which allows for the decomposition of productivity change into both a movement of the economy towards its potential and the change of the latter. Hence, this model distinguishes between efficiency change and technical change. The frontier approach determines the weight of respective production factors endogenously, and therefore does not require any data on input shares or prices. In this way, the questionable assumption of perfectly competitive factor markets is not required. Like growth accounting, our model allows us to impute productivity growth to different production factors. Furthermore, in contrast to growth accounting it is able to model multiple input/multiple output production processes.

In the proposed model, the economy is represented by the Leontief input-output model, which is extended by the constraints of primary inputs. Using the multi-objective optimization model the efficiency frontier of the economy is generated. Its solutions define efficient virtual decision making units (DMUs). The efficiency of the given economy is defined as the difference between the potential of an economy and its actual performance and can be obtained as a solution of a DEA model. A Luenberger productivity indicator is proposed to estimate productivity change over time; this is then decomposed in a way that enables one to examine the contributions of individual production factor and individual outputs to productivity change. The results allow the inference of which inputs or outputs of an economy are the drivers of the overall productivity change. Total factor productivity change (TFPCH), in turn, is decomposed into efficiency change (EFFCH, catch-up) and technical change (TECHCH, frontier shift) components.

For the purposes of illustration, the proposed approach is then used to estimate the long-term total factor productivity growth in the United States of America for the period 1977 to 2006. Here, a clear average productivity growth of 24.6 percent is observed, with primary inputs and final demand contributing 12.3 percent each. Among the primary inputs considered, high-skilled labor shows the highest positive contribution and capital the highest negative contribution. Services together with trade, transport & utilities contribute the most of all commodities. All commodities contribute positively to productivity growth, with the exception of manufacturing. A closer look at the components of TFPCH reveals technical progress as the main source. Technical progress is mostly driven by capital whereas low-skilled labor contributes negatively. Furthermore, we also find a slight regress in efficiency, which is mainly driven by a decrease in the utilization of capital.

The construction of the efficiency frontier permits an assessment of the economies' actual performance with respect to its own potential (even in the case of multiple outputs and inputs) without the need to compare it with other economies – economies that may possess different technologies and varying mutual interdependencies due to international trade. Following our results, the relative merits of both approaches (conventional frontier approach and neoclassical growth accounting) can be utilized. For inter-temporal comparisons of productivity the movement of the economy towards the frontier and its shift can be obtained by using the DEA formulation.

Finally, we point to possible avenues for future research to take. Firstly, in order to ensure Pareto-Koopmans' efficient solution, slacks should be taken into account in the efficiency analysis. This could be achieved by adopting a directional slacks-based measure approach in spirit of both Fukuyama and Weber (2009) and Mahlberg and Sahoo (2011). Secondly, the model could be enlarged by the addition of data on pollution (e.g. CO_2 emissions) and pollution abatement, in order to measure eco-efficiency change and eco-productivity change over time.

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