Department of Economic Policy
Working Paper Series

WP No. 2

Data Envelopment Analysis for Measuring of Economic Growth in Terms of Welfare Beyond GDP

Author: Lábaj, M. – Luptáčik, M. – Nežinský, E.

Date: January 20, 2013

Published by:

University of Economics in Bratislava, Department of Economic Policy, Dolnozemská cesta 1, 852 35 Bratislava

Periodicity: irregular

Peer-reviewed

ISSN 1339 - 0430

Data Envelopment Analysis for Measuring of Economic Growth in Terms of Welfare Beyond GDP[1]

Lábaj, M. – Luptáčik, M. – Nežinský, E.

Department of Economic Policy, Faculty of National Economy, University of Economics in Bratislava

Abstract
Recent discussions on the definition of growth in terms of welfare beyond GDP suggest that it is of urgent need to develop new approaches for measuring the economic performance of the firms and national economies. The new concepts should take into account simultaneously economic as well as social and environmental goals. We first discuss several approaches to productivity measures. Then we extend the Data Envelopment Analysis models for environment to measure the so called eco-efficiency and for social indicators to take into account the social performance. For an illustration, we perform the analysis of 30 European countries in the year 2010. In the last section we discuss the possibilities of inter-temporal analysis of proposed models and of their use in ex-ante evaluation of different policy scenarios.

Keywords: eco-efficiency, data envelopment analysis, beyond GDP

JEL codes: C43, C61, O47

1 Introduction and motivation

Recent developments in the discussion on the definition of growth in terms of welfare beyond GDP suggest that it is of urgent need to develop and to use new approaches for measuring the economic performance of the firms and national economies. The new concepts should take into account simultaneously economic as well as social and environmental goals.
One of these new concepts is eco-efficiency, first introduced by Schaltegger and Sturm (1989) and defined as ratio between environmental impact added and value added. The World Businesses Council for Sustainable Development (WBCSD) defines eco-efficiency as "…the delivery of competitive-priced goods and services, that satisfy human needs and bring quality of life, whilst progressively reducing ecological impacts and resource intensity throughout the lifecycle, to a level at least in line with the Earth's estimated carrying capacity" (WBCSD 1966). The European Union recognizes eco-efficiency as a key option to reach the Lisbon competitiveness (European Commission, 2005). Strengthening eco-efficiency has also been identified by the OECD as one of the major strategic elements in its work on sustainability (OECD, 1998).
Eco-efficiency as a common denominator incorporating different outcomes of economic activities (production of undesirable goods jointly with desirable goods) aims at achieving more goods and services with fewer resources as well as less waste and emissions. Therefore we need models taking into account simultaneously multiple outputs and multiple inputs where some of the outputs (undesirable outputs like the waste and emissions) are not given in monetary units and therefore do not allow to be aggregated by one monetary value. These difficulties can be overcome by using Data envelopment analysis (DEA), as a non-parametric production-frontier

[1] This working paper is a part of research project VEGA 1/0906/12: „Technological Change, Catch-Up and Ecoefficiency: Growth and Convergence in EU Countries".

methodology and by multiple criteria analysis (MCA). In this way multiple goals of economic policy can be taken into account simultaneously and the potential trade-offs systematically explored.

The paper is organized as follows. In its first part the extensions of DEA models for measuring the eco-efficiency taking into account indicators for the social performance (welfare, equity, and other indicators as proposed e. g. in the Stiglitz report) are presented. We start in Section 2 with the discussion of the different approaches to productivity analysis and then present extended DEA-models for environmental and social aspects. Various models that allow us to approach efficiency and to identify the sources of inefficiency in different ways are developed. In Section 3, for illustration purposes, the eco-efficiency and socio-economic efficiency of 30 European countries (27 EU countries + Iceland, Norway and Switzerland) in the year 2010 is analyzed.

In the last part of the paper we discuss several possibilities for further research and work. In Section 4 the inter-temporal comparison of the eco-efficiency, as proposed in Mahlberg – Luptáčik – Sahoo (2011) allows to identify the drivers of total productivity change and to examine the contributions of economic and environmental factors towards the total factor productivity change. It is the merit of DEA as the non-parametric production-frontier methodology that the change in the eco-efficiency can be decomposed into efficiency change (catch-up) and technical change (frontier shift) components. In this way, the analysis could be extend for models with economic, environmental and social aspects and the results could provide important insights into determinants of economic growth incorporating economic, environmental and social goals.

The last Section 5 deals with the question how the proposed DEA models can be used in evaluation of different policy scenarios (ex-ante analysis) as shown in the paper by Bosetti – Buchner (2009). Introducing the weight restrictions into DEA models allows us to examine the impact of the changes in the objective priorities of the policy makers on the social welfare.

2 Welfare Beyond GDP – methodological aspects

In this part we first discuss various approaches to productivity analysis and then develop several models for the analysis of efficiency, taking into account environmental as well as social aspects of economic development.

2.1 Different approaches to productivity analysis

There are two approaches to productivity analysis in the literature. The first one is a neoclassical approach which goes back to Solow´s seminal paper (1957) and that was later developed by Griliches, Jorgenson and Abramovitz among others (for an historical overview see Griliches, 1995). It is based on standard neoclassical production function and through the so called growth accounting methodology decomposes the output growth into the contribution of growth in inputs and the contribution of residual that is referred as productivity growth. Besides the restriction of assumptions behind the production function specification described e.g. in Acemoglu (2009, chapter 2), neoclassical growth accounting does not distinguish between efficiency change and technical change, and is not able to model multiple input/multiple output production processes. The second one, frontier approach, can be implemented by different techniques such as mathematical programming techniques or econometric techniques. Econometric techniques in frontier analysis are usually referred to as Stochastic Frontier Approach and are restricted to single output production. Mathematical programming approach is known as Data Envelopment Analysis and is suitable for a multiple input/multiple output production system analysis. Other differences between these two techniques boil down to two essential features. The econometric approach is stochastic. This enables it to attempt to distinguish the effects of noise from those of inefficiency, thereby providing the basis for statistical inference. The programming approach is nonparametric. This enables it to avoid confounding the effects of misspecification of the functional form with those of inefficiency (Fried et al, 2008, p. 33) In contrast to neoclassical growth accounting DEA needs no factor price information and no equilibrium assumption necessary to equate price and marginal product. The weights required for aggregation of inputs and outputs are obtained as an integral part of the optimization process.

Although each approach tracks changes in the output-input ratio, the analytical implications are quite distinct. The neoclassical approach imputes productivity growth to factors of production, but cannot distinguish a movement towards the production possibilities frontier and a movement of the frontier. The frontier approach allows decomposing productivity growth into a movement of the economy towards the efficiency frontier and a shift of the frontier. Productivity change is equal to efficiency change plus technical change. The frontier approach, however, is not capable of imputing value to factor inputs. In the paper by ten Raa and Mohnen (2002) a synthesis of both approaches is provided. They estimated total factor productivity (TFP) growth without recourse to data on factor input prices. In their work they reproduced the neoclassical TFP growth formulas, but within a framework and in the spirit of Data Envelopment Analysis.

Another advantage of DEA is that it enables us to measure efficiency in a system with inputs and output in different units. It does not require having all the variables expressed in monetary units and thus extend the analysis for environmental and social aspects that are usually not expressed in monetary terms. In this way, we can move toward the new concept and measurement of welfare beyond GDP and evaluate different development scenarios as well.

2.2 Extended DEA-models for environmental and social aspects

One of these new concepts is eco-efficiency that considers inputs, desirable outputs and undesirable outputs in one model and takes economic as well as ecological aspects simultaneously into account. The main problem in developing of the eco-efficiency indicators is the lack of monetary evaluations like market prices for the undesirable outputs (the waste and emissions). As already mentioned DEA is able to use the data in different units and therefore can provide an appropriate methodology for measuring of the eco-efficiency. There is a extensive literature on DEA models when some outputs are undesirable, starting with the seminal paper by Färe et al (1989) and following by Färe at al (1996), Tyteca (1996), Tyteca (1997), Dyckhoff – Allen (2001) and others.

Korhonen and Luptacik (2004) proposed different variants of DEA models for the evaluation of eco-efficiency in a single period. The first model (denoted "Model A") uses negative weights for undesirable outputs, the second (denoted "Model B") takes the undesirable outputs as inputs, and in the "Model C" negative weights are used for the inputs. They show that the set of (strongly) efficient decision making units (DMUs) is the same for all the models. However, the different variants provide deeper insights into the underlying sources of eco-efficiency differential across DMUs and therefore show different ways of increasing eco-efficiency.

New approach for the analysis of economic performance of nations to a wider perspective of social welfare, encompassing growth in GDP as well as the changes in the distribution of income proposed Rao and Coelli (1999). They use a non-parametric method to measure productivity growth in different countries and generalize the approach to include both inequality and level of income as joint determinants of total welfare resulting from economic activity.

In this part, we follow the models in Korhonen and Luptacik (2004) and extend them for social aspects as well.

In a standard DEA model we assume to have n decision making units (DMUs) each employing m inputs to produce p outputs. The observed non-negative measures of input and output make up input matrix $\mathbf{X} \in \Re_+^{m \times n}$ and output matrix $\mathbf{Y} \in \Re_+^{p \times n}$. Desirable outputs correspond to indices 1, 2, ..., k while undesirable ones are indexed from $k+1$ to p. Matrix $\mathbf{Y}$ can be decomposed into two parts: $\mathbf{Y}^{\mathbf{g}} \in \Re_+^{p \times n}$ containing desirable outputs and $\mathbf{Y}^{\mathbf{b}} \in \Re_+^{(p-k) \times n}$ containing "bads" (undesirable outputs).Vectors of desirable and undesirable outputs of unit *j* are referred to by $\mathbf{y}_j^{\mathbf{g}}$ and $\mathbf{y}_j^{\mathbf{b}}$ respectively. $[1, ..., 1]^T$ is denoted **1**.

Classical DEA defines the measure of *technical* efficiency of a DMU as a ratio of a weighted sum of desirable outputs to a weighted sum of inputs (virtual outputs / virtual inputs) such that no DMU´s efficiency score can exceed one The basic Charnes - Cooper – Rhodes (1978) input-oriented model (denoted further as CCR-model) measures the technical efficiency of a DMU ´0´ by employing following fractional program:

MODEL I

$$\max \quad h_0(\boldsymbol{\mu}, \mathbf{v}) = \frac{\sum_{r=1}^{k} y_{r0}\mu_r}{\sum_{i=1}^{m} x_{i0}v_i} \tag{1}$$

$$\text{s.t.} \quad \frac{\sum_{r=1}^{k} y_{rj}\mu_r}{\sum_{i=1}^{m} x_{ij}v_i} \leq 1 \qquad (j = 1, 2, ..., n)$$

$$u_r \geq \varepsilon,\ v_i \geq \varepsilon \qquad (r = 1, 2, ..., k), \quad (i = 1, 2, ..., m)$$

ε > 0 („Non-Archimedean“)

Using the transformation proposed in Charnes – Cooper (1962) the problem (1) can be transformed into the following linear programming problem:

Model I*: primal and dual linear program*

$$\begin{aligned} \min \quad & g_I = \theta - \mathbf{1}^T\mathbf{s}^- - \mathbf{1}^T\mathbf{s}^+ \\ \text{s.t.} \quad & \theta\mathbf{x_0} - \mathbf{X}\boldsymbol{\lambda} - \mathbf{s}^- = \mathbf{0} \\ & \mathbf{Y}\boldsymbol{\lambda} - \mathbf{y_0} - \mathbf{s}^+ = \mathbf{0} \\ & \boldsymbol{\lambda}, \mathbf{s}^+, \mathbf{s}^- \geq 0 \end{aligned} \tag{2a}$$

$$\begin{aligned} \max \quad & h_I = \mathbf{u}^T\mathbf{y_0} \\ \text{s.t.} \quad & \mathbf{u}^T\mathbf{Y} - \mathbf{v}^T\mathbf{X} \leq 0 \\ & \mathbf{v}^T\mathbf{x_0} = 1 \\ & \mathbf{u}, \mathbf{v} \geq \varepsilon\mathbf{1} \end{aligned} \tag{2b}$$

Each DMU solves the optimization problem (2) in order to assign most favourable weights **u** to its outputs and **v** to its inputs. If the resulting score θ is equal 1 and all slack variables are equal zero, the DMU is defined as *efficient* and becomes part of the *efficiency frontier* .θ smaller than one , can be interpreted as need for radial reduction of inputs - keeping the outputs unchanged - to project itself onto the efficiency frontier. In other words DMUs controls its inputs to increase the efficiency.

As proposed by Korhonen – Luptáčik (2004), the evaluation of eco-efficiency can be addressed by decomposing the problem in two parts. First, **Model I** is set up to measure *technical efficiency* in a standard way described by the fractional program (1). Additionally a new model taking into account the ratio of a weighted sum of desirable outputs to a weighted sum of undesirable outputs ("bads") - denoted as **Model II** and computing *ecological efficiency* - can be formulated:

MODEL II

$$\max \quad h_{II} = \frac{\sum_{r=1}^{k} y_{r0}\mu_r}{\sum_{s=k+1}^{p} y_{s0}\mu_s} \tag{3}$$

$$\text{s.t.} \quad \frac{\sum_{r=1}^{k} y_{rj}\mu_r}{\sum_{s=k+1}^{p} y_{sj}\mu_s} \leq 1 \qquad (j = 1, 2, ..., n)$$

$$\mu_r \geq \varepsilon, \mu_s \geq \varepsilon \qquad \begin{array}{l}(r = 1, 2, ..., k)\\(s = k+1, ..., p)\end{array}$$

$\varepsilon > 0$ („Non-Archimedean“)

The corresponding linear programming problem takes the following form:

Model II: *primal and dual linear program*

$$\begin{aligned} \min \quad & g_{II} = \theta - \mathbf{1}^{\mathrm{T}}\mathbf{s}^{\mathbf{b}} - \mathbf{1}^{\mathrm{T}}\mathbf{s}^{\mathbf{g}} \\ \text{s.t.} \quad & \theta\mathbf{y}_{\mathbf{0}}^{\mathbf{b}} - \mathbf{Y}^{\mathbf{b}}\boldsymbol{\lambda} - \mathbf{s}^{\mathbf{b}} = \mathbf{0} \\ & \mathbf{Y}^{\mathbf{g}}\boldsymbol{\lambda} - \mathbf{y}_{\mathbf{0}}^{\mathbf{g}} - \mathbf{s}^{\mathbf{g}} = \mathbf{0} \\ & \boldsymbol{\lambda}, \mathbf{s}^{\mathbf{g}}, \mathbf{s}^{\mathbf{b}} \geq 0 \end{aligned} \tag{4a}$$

$$\begin{aligned} \max \quad & h_{II} = \mathbf{u}_{\mathbf{g}}^{\mathrm{T}}\mathbf{y}_{\mathbf{0}}^{\mathbf{g}} \\ \text{s.t.} \quad & \mathbf{u}_{\mathbf{g}}^{\mathrm{T}}\mathbf{Y}^{\mathbf{g}} - \mathbf{u}_{\mathbf{b}}^{\mathrm{T}}\mathbf{Y}^{\mathbf{b}} \leq 0 \\ & \mathbf{u}_{\mathbf{b}}^{\mathrm{T}}\mathbf{y}_{\mathbf{0}}^{\mathbf{b}} = 1 \\ & \mathbf{u}_{\mathbf{g}}, \mathbf{u}_{\mathbf{b}} \geq \varepsilon\mathbf{1} \end{aligned} \tag{4b}$$

Indicators from both models are then used in the new DEA output-oriented model as the outputs with inputs equalling 1. In this way, the indicator for eco-efficiency is provided.

Taking the undesirable and desirable outputs simultaneously into account different models can be used. Two alternatives are represented by the following models.

The first model denoted by A is the model with negative sign for undesirable outputs (bads). In this model the DMUs control the inputs. In order to increase the eco-efficiency, the DMUs will reduce proportionally the inputs. he optimization problem reads as:

MODEL A

$$\max \quad h_A = \frac{\sum_{r=1}^{k} \mu_r y_{r0} - \sum_{s=k+1}^{p} \mu_s y_{s0}}{\sum_{i=1}^{m} v_i x_{i0}} \tag{5}$$

$$\text{s.t} \quad \frac{\sum_{r=1}^{k} \mu_r y_{rj} - \sum_{s=k+1}^{p} \mu_s y_{sj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1$$

$j = 1, 2, ..., n$; μ_r, $v_i \ge \varepsilon$; $r = 1, 2, ..., p$; $i = 1, 2, ..., m$;

$\varepsilon > 0$ („Non-Archimedean“)

Using Charnes - Cooper transformation, the following pair of primal and dual linear programs (6a) and (6b) as the modification of CCR model (Charnes et al., 1978, 1979) comes out:

Model A*: primal and dual linear program*

$$\begin{aligned} \min \quad & g_A = \theta - \varepsilon \mathbf{1}^T (\mathbf{s^b} + \mathbf{s^g} + \mathbf{s}^-) \\ \text{s.t.} \quad & \mathbf{Y^g}\boldsymbol{\lambda} - \mathbf{s^g} = \mathbf{y_0^g} \\ & \mathbf{Y^b}\boldsymbol{\lambda} + \mathbf{s^b} = \mathbf{y_0^b} \\ & \mathbf{X}\boldsymbol{\lambda} - \theta \mathbf{x_0} + \mathbf{s}^- = \mathbf{0} \\ & \boldsymbol{\lambda}, \mathbf{s^g}, \mathbf{s^b}, \mathbf{s}^- \ge \mathbf{0} \end{aligned} \tag{6a}$$

$$\begin{aligned} \max \quad & h_A = \mathbf{u_g^T y_0^g} - \mathbf{u_b^T y_0^b} \\ \text{s.t.} \quad & \mathbf{u_g^T Y^g} - \mathbf{u_b^T Y^b} - \mathbf{v^T X} \le 0 \\ & \mathbf{v^T x_0} = 1 \\ & \mathbf{u_g}, \mathbf{u_b}, \mathbf{v} \ge \varepsilon \mathbf{1} \end{aligned} \tag{6b}$$

In the **Model B**, undesirable outputs are considered as inputs. This approach results in the following optimization problem:

MODEL B

$$\max \quad h_B = \frac{\sum_{r=1}^{k} \mu_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0} + \sum_{s=k+1}^{p} \mu_s y_{s0}} \tag{7}$$

$$\text{s.t} \quad \frac{\sum_{r=1}^{k} \mu_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij} + \sum_{s=k+1}^{p} \mu_s y_{sj}} \le 1$$

$j = 1, 2, ..., n$; μ_r, $v_i \ge \varepsilon$; $r = 1, 2, ..., p$; $i = 1, 2, ..., m$;

$\varepsilon > 0$ („Non-Archimedean“)

Applying transformation of variables to (7) yields a pair of primal – dual programs (8a) and (8b).

Model B*: primal and dual linear program*

$$\begin{aligned} \min \quad & g_B = \theta - \varepsilon \mathbf{1}^T(\mathbf{s^b} + \mathbf{s^g} + \mathbf{s^-}) \\ \text{s.t.} \quad & \mathbf{Y^g\lambda} - \mathbf{s^g} = \mathbf{y_0^g} \\ & \mathbf{Y^b\lambda} - \theta\mathbf{y_0^b} + \mathbf{s^b} = \mathbf{0} \\ & \mathbf{X\lambda} - \theta\mathbf{x_0} + \mathbf{s^-} = \mathbf{0} \\ & \boldsymbol{\lambda}, \mathbf{s^g}, \mathbf{s^b}, \mathbf{s^-} \geq \mathbf{0} \end{aligned} \qquad (8a)$$

$$\begin{aligned} \max \quad & h_B = \mathbf{u_g^T y_0^g} \\ \text{s.t.} \quad & \mathbf{u_g^T Y^g} - \mathbf{u_b^T Y^b} - \mathbf{v^T X} \leq 0 \\ & \mathbf{v^T x_0} + \mathbf{u_b^T y_0^b} = \mathbf{1} \\ & \mathbf{u_g}, \mathbf{u_b}, \mathbf{v} \geq \varepsilon\mathbf{1} \end{aligned} \qquad (8b)$$

In this model, attempt to increase eco-efficiency requires simultaneous reduction of both inputs and undesirable outputs. It can be shown that the eco-efficiency indicator obtained by Model A cannot be greater than efficiency score from Model B.

For the purpose of our analysis, it is useful to apply the models in output orientation. Then the resulting score indicates the changes in outputs needed to reach efficiency frontier, in other words, we control outputs as the goals of economic policy. One can reasonably argue that inputs like capital stock cannot be easily changed within a short time period. Since the size of DMUs under consideration exhibits quite large variability we relax the assumption of constant returns to scale. In our modelling, it is made by adding scalar μ_0 to objective function as proposed in Banker et al. (1984). This can be used to measure pure technical efficiency with a variable returns to scale (VRS) assumption and the model is called BCC-O to refer both to the VRS (the approach proposed by Banker – Charnes – Cooper) and orientation of the model. In our analysis, we apply BCC-O while adopting the approach to treating undesirable outputs of models A and B as well. The output-oriented Model A extended by the constraint implying variable returns to scale takes the following form of primal – dual linear programs (after Charnes – Cooper transformation of variables mentioned above):

Model A(O): *primal and dual program*

$$\begin{aligned} \max \quad & g_{A(O)} = \theta + \varepsilon \mathbf{1}^T(\mathbf{s^b} + \mathbf{s^g} + \mathbf{s^-}) \\ \text{s.t.} \quad & \mathbf{Y^g\lambda} - \mathbf{s^g} = \theta\mathbf{y_0^g} \\ & \mathbf{Y^b\lambda} + \mathbf{s^b} = \theta\mathbf{y_0^b} \\ & \mathbf{X\lambda} + \mathbf{s^-} = \mathbf{x_0} \\ & \mathbf{1}^T\boldsymbol{\lambda} = 1 \\ & \boldsymbol{\lambda}, \mathbf{s^g}, \mathbf{s^b}, \mathbf{s^-} \geq 0 \end{aligned} \qquad (9a)$$

$$\begin{aligned} \min \quad & h_{A(O)} = \mathbf{v^T x_0} + \mathrm{u_0} \\ \text{s.t.} \quad & \mathbf{u_g^T Y^g} - \mathbf{u_b^T Y^b} - \mathbf{v^T X} + \mathbf{1}^T \mathrm{u}_0 \leq 0 \\ & \mathbf{u_g^T y_0^g} - \mathbf{u_b^T y^b} = 1 \\ & \mathbf{u_g}, \mathbf{u_b}, \mathbf{v} \geq \varepsilon\mathbf{1} \end{aligned} \qquad (9b)$$

In the similar way we re-formulate Model B into BCC-O to get primal and dual of Model B(O). The difference between Model A(O) and Model B(O)is, that though both are output-oriented the former requires simultaneously augmenting both "goods" and "bads" while the latter indicates how to increase desirable outputs only in order to reach efficiency frontier.

Model B(O): *primal and dual program*

$$\max \quad g_{B(O)} = \theta + \varepsilon \mathbf{1}^T(\mathbf{s^b} + \mathbf{s^g} + \mathbf{s}^-)$$
$$\text{s.t.} \quad \mathbf{Y^g}\boldsymbol{\lambda} - \mathbf{s^g} = \theta \mathbf{y_0^g}$$
$$\mathbf{Y^b}\boldsymbol{\lambda} + \mathbf{s^b} = \mathbf{y_0^b} \qquad (10a)$$
$$\mathbf{X}\boldsymbol{\lambda} + \mathbf{s}^- = \mathbf{x_0}$$
$$\mathbf{1}^T\boldsymbol{\lambda} = 1$$
$$\boldsymbol{\lambda}, \mathbf{s^g}, \mathbf{s^b}, \mathbf{s}^- \geq 0$$

$$\min \quad h_{B(O)} = \mathbf{u_g}^T\mathbf{y_0^b} + \mathbf{v}^T\mathbf{x_0} + u_0$$
$$\text{s.t.} \quad \mathbf{u_g}^T\mathbf{Y^g} - \mathbf{u_b}^T\mathbf{Y^b} - \mathbf{v}^T\mathbf{X} + \mathbf{1}^T u_0 \leq 0$$
$$\mathbf{u_g}^T\mathbf{y_0^g} - \mathbf{u_b}^T\mathbf{y^b} = 1 \qquad (10b)$$
$$\mathbf{u_g}, \mathbf{u_b}, \mathbf{v} \geq \varepsilon \mathbf{1}$$

In order to provide new indicators for measuring of the economic performance of the national economies in terms of welfare beyond GDP, we extent the DEA models by introducing of new variables.
We start with standard output-oriented DEA model measuring the pure technical efficiency by taking capital (K) and labour (L) as inputs and GDP (Y) as output (further denoted as **Model 1)**. Capital and labour are included in each of these models as inputs. Then in **Model 2** we included greenhouse gases (GHG) emissions as additional output taking with negative sign (see Model A).

Model 3 is an extension of **Model 1** by adding of the Gini index as the income inequality indicator. Using the Gini index as an output variable we transformed it by subtracting from one (denoted further as Gini1). The higher value of Gini1 (in other words lower Gini index) means better performance with respect to social welfare.

In **Model 4** we included all variables simultaneously in a new model such that the overall efficiency index exploits measures of undesirable outputs by putting GHG among inputs as well as Gini1 to outputs. Output orientation means that resulting score should be interpreted in terms of changing the both output indicators (GDP as an economic indicator and income inequality as a social indicator) in order to improve "eco-social welfare" performance. In this way the preferences of policy makers can be taken into account. The approach based at the adding of the new variables allows identify the impact of the given indicators for the overall performance of the particular DMU.

The other approach was carried out by computing ecological scores using **Model I** and **II.** In **Model 5**, we compute ecological efficiency as described by (3) treating data on GDP as desirable outputs and data on GHG as "bads". **Model 6** is formulated in the same way as **Model 3** and includes both economic performance and social equality measures. Further, we use efficiency scores from **Model 5** and **Model 6** as outputs with inputs equal 1 in the composed model to obtain the optimal "*socio-eco-efficiency*" score.

An overview of the models and variables is provided in the following Table 1.

Table 1: Overview of employed models

No.	Specification *	Description	Inputs			Outputs	
1.	*Model I(O)*	Technical efficiency	capital	labor		GDP	
2.	*Model A(O)*	Eco-efficiency	capital	labor		GDP	Emissions (-)
3.	*Model B(O)*	Income inequality-adjusted efficiency	capital	labor		GDP	1 - Gini
4.	*Model B(O)*	Social welfare efficiency	capital	labor	emissions	GDP	1 - Gini
5.	*Model II*	Ecological efficiency			emissions	GDP	
6.	*Model III*	Income inequality-adjusted efficiency	capital	labor		GDP	1 - Gini
7.	*Model I(O)*	Social welfare efficiency			1	Score Model I	Score Model II

* All models are BCC-O

Because DEA can used the inputs and outputs measured in different units these models can be augmented both on the input and output side by adding more indicators specifying areas of interest as ecology or social welfare more precisely. Detailed data on various emissions such as methan, SO_x, NO_x or living conditions such as life expectancy, number of persons endangered by poverty and so on may be used. Human capital may be added to standard inputs.

3 Empirical analysis

In order to show the possibilities and potential of the proposed approach for measuring of economic growth and social welfare we analyse the performance of 30 European countries (EU-27 countries plus Iceland, Norway and Switzerland).

3.1 Data

Though DEA allows for multiple-input and multiple-output handling, we consider two most important inputs – capital stock and labour - which are employed by national economies (DMUs) to produce GDP as the desirable output and greenhouse gases emissions as the undesirable output. As already mentioned it is easily possible to extend the models by adding more indicators both to inputs and outputs. In this study, the “bads” have been determined so as to reflect negative impact on the social welfare. Income inequality describing another negative impact for the social welfare is measured by Gini index. We use EUROSTAT data of the year 2010 to measure GDP and capital stock in PPS, labour in thousands of persons employed, GHG in thousands of tons. Initial estimates of capital in EU-30 in national currencies are taken from Dujava (2012) that computed the capital/output shares taking into account the specifics of the new EU member states. Table 2 provides statistical overview of the data used. We took GDP data at current prices since no intertemporal analysis will be carried out in this study.

Table 2: Descriptive statistics of Input/Output Data

	Capital	Labor	Emissions	GDP	Gini1
Max	7557986	40603	936544	2371033	76
Min	20819	167	3035	8628	63
Average	1343228	7676	161119	426524	71
St. dev.	1903663	10043	217224	600219	4

- No. of DMUs = 30

3.2 Results and discussion

In the first column of Table 3 the results for technical efficiency with capital and labour as the inputs and GDP as the output are presented. Among the nine efficient countries are countries with a very strong economy (like Germany, Norway, United Kingdom) and some small countries (like Iceland, Malta, Luxemburg), due to variable returns to scale DEA modeling. The lowest efficiency scores are achieved in Estonia, Latvia, Czech Republic and Slovenia. The efficiency score for Slovakia (0.75) means that –under the given labour and capital inputs - Slovakia should increase GDP by 25% to become efficient, which indicates significant potential for improving of the technical efficiency.

Table 2: Efficiency scores of European countries for the year 2010 (Models 1-4)

Model 1 *Technical Efficiency*		Model 2 *Eco-efficiency*		Model 3 *Social efficiency*		Model 4 *Social Eco-efficiency*	
DMU	Score	DMU	Score	DMU	Score	DMU	Score
Norway	1	Switzerland	1	Norway	1	Switzerland	1
Belgium	1	Belgium	1	Belgium	1	Belgium	1
Iceland	1	Norway	1	Iceland	1	Norway	1
United Kingdom	1	Iceland	1	United Kingdom	1	Iceland	1
Denmark	1	Denmark	1	Denmark	1	Denmark	1
Germany	1	Germany	1	Germany	1	Germany	1
Malta	1	United Kingdom	1	Finland	1	United Kingdom	1
Luxembourg	1	Malta	1	Slovenia	1	Sweden	1
France	1	Luxembourg	1	Malta	1	Finland	1
Poland	0.993	France	1	Luxembourg	1	Slovenia	1
Italy	0.961	Poland	0.993	France	1	France	1
Netherlands	0.959	Italy	0.961	Sweden	0.996	Malta	1
Finland	0.950	Netherlands	0.959	Netherlands	0.994	Luxembourg	1
Spain	0.934	Sweden	0.953	Hungary	0.994	Netherlands	0.994
Ireland	0.908	Finland	0.950	Poland	0.993	Hungary	0.994
Switzerland	0.881	Spain	0.934	Czech Republic	0.983	Poland	0.993
Sweden	0.880	Ireland	0.908	Slovakia	0.972	Czech Republic	0.983
Portugal	0.859	Portugal	0.874	Italy	0.970	Slovakia	0.972
Austria	0.851	Austria	0.851	Austria	0.969	Italy	0.970

Cyprus	0.841	Cyprus	0.841	Cyprus	0.952	Austria	0.969
Lithuania	0.794	Lithuania	0.794	Spain	0.934	Cyprus	0.952
Slovakia	0.753	Slovakia	0.753	Switzerland	0.925	Spain	0.934
Greece	0.734	Greece	0.734	Estonia	0.914	Estonia	0.914
Romania	0.708	Romania	0.708	Ireland	0.908	Ireland	0.908
Bulgaria	0.705	Bulgaria	0.705	Portugal	0.895	Portugal	0.896
Hungary	0.700	Hungary	0.700	Greece	0.880	Greece	0.880
Slovenia	0.686	Slovenia	0.686	Bulgaria	0.878	Bulgaria	0.878
Czech Republic	0.648	Czech Republic	0.648	Romania	0.875	Romania	0.875
Latvia	0.619	Latvia	0.619	Lithuania	0.851	Lithuania	0.851
Estonia	0.596	Estonia	0.596	Latvia	0.848	Latvia	0.849

With **Model 2** we measure the eco-efficiency and its results are given in the second column of Table 1. Compared to **Model 1,** greenhouse emissions as an undesirable output into the model are included. Otherwise inputs (capital and labor) and output (GDP) remain the same. So, the efficiency scores for **Model 2** cannot be lower than in **Model 1**. There are three countries that improved their score – Switzerland, Sweden and Portugal due to their strength in environmental performance. Switzerland, with technical efficiency score 0,881 in Model 1 is now eco-efficient as other nine–already technical efficient – countries. The eco-efficiency score for Slovakia (0.75) does not changed compared with the technical efficiency which indicates relative weakness in the environmental performance.

Social aspects of economic development are included in **Model 3** that extends the **Model 1** for an indicator of income equality as a new output. Relatively small differences in income inequality among the countries (see Table 2) are reflected in the results of the **Model 3** as well. Low income inequality in Finland (Gini1 is equal 74,6) and Slovenia (with second highest Gini1 coefficient equal 76,2; Norway with 76,4 is the best country) is reflected in their position on efficiency frontier in **Model 3**. Sweden (with already high technical efficiency and high eco-efficiency) is among the countries that improved the efficiency score significantly as well. Due to the relative high Gini1 index for Slovakia, the income inequality adjusted efficiency score for Slovakia improved to 0.97. Taking into account income inequality Slovakia should increase the GDP and Gini1 by 3% in order to be efficient.

Model 4 takes into account economic, environmental and social indicators simultaneously. With comparison to **Model 1** there are 13 efficient countries in **Model 4**. Because the efficiency score cannot be lower than in previous models there are the same nine countries as in **Model 1,** Switzerland that is efficient in **Model 2** and Finland together with Slovenia from **Model 3**. The new efficient country is Sweden, although it was not efficient in the previous models describing particular areas of interest as high environmental quality or social inequality-related economic performance. In those models, some countries were able to weight their indicators according their strengths so as to maximize the efficiency score. In the model employing the indicators simultaneously the relative high performance of Sweden from economic (technical efficiency 0.88), ecological (eco-efficiency 0.99) as well as social welfare point of view results in the overall efficiency. The result for Sweden shows that the efficiency is based on the well performance with respect to the all (economic, environmental as well as social) indicators.

Deeper insights into strengths and weakness of particular economies and into potentials for improving the efficiency can be obtained looking the multipliers for the inputs ant outputs which are provided by solutions of the dual problems (2b), (9b) and corresponding dual problems to the models in Table 1.

For illustration we chose six countries: Germany as country with very strong economy, Greece, country with weak economic performance, Finland, Sweden and Austria, countries with high standard of living and Slovakia as country with rapid economic development.

Table 3: Score and weighted data from Model 4 for chosen European economies

		Inputs			Outputs	
DMU	**Score**	**Capital**	**Labor**	**Emissions**	**GDP**	**Inequality**
Germany	1	0,229	0,662	0	1	0
Finland	1	0,204	0,039	0	0,240	0,760
Sweden	1	0,301	0	0,177	0,535	0,465
Slovakia	0,972	0,002	0	0	0	1
Austria	0,969	0	0	0	0,009	0,991
Greece	0,880	0	0	0	0,009	0,991

In Table 4 we can see the score and weighted data for chosen countries. Weighted data on outputs are obtained by multiplication of the outputs by the corresponding multipliers. They give us an indication of the importance of given output for the efficiency score of the particular countries. As follows from constraints (9b) and (10b), sum of weighted outputs must be equal to one. Every country can choose its weights (multipliers) according their strengths such their efficiency will be maximized. The results in Table 4 shows very strong economic performance of Germany compared to other countries in the sample (the strongest economy in the EU) with the weighted data 1 for GDP and zero for inequality indicator.

On the other side, the main contribution to the efficiency score for Slovakia is provided by the inequality indicator where the position of the country is much better-compared to other countries in the sample-than in technical efficiency. Similar result can be found for Austria. The weakness of Greece in technical efficiency is not surprising.

The results for Sweden and Finland show more balanced contributions of both output factors and confirm that the welfare in these countries is based on all indicators: economic, environmental as well as social In Sweden, both indicators contributed to the evaluation approximately by the same size while in Finland played the inequality higher importance.

Finally, the projections of the chosen countries to the efficiency frontier as presented in Table 5 shows the possibilities for improvement of efficiency.

Table 4: Projections of chosen European economies to the efficiency frontier from Model 4

DMU	1/Score				DMU	1/Score			
I/O	Data	Projection	Difference	%	I/O	Data	Projection	Difference	%
Germany	1				**Greece**	1,136			
Capital	7557986,3	7557986,3	0,0	**0,00%**	**Capital**	888982,4	816488,8	-72493,6	**-8,15%**
Labor	40603,0	40603,0	0,0	**0,00%**	**Labor**	4711,7	3611,5	-1100,2	**-23,35%**
Emissions	936544,0	936544,0	0,0	**0,00%**	**Emissions**	118287,0	77389,0	-40898,0	**-34,58%**
GDP	2371033,2	2371033,2	0,0	**0,00%**	**GDP**	241981,7	274973,0	32991,3	**13,63%**
Inequality	70,7	70,7	0,0	**0,00%**	**Inequality**	67,1	76,2	9,1	**13,63%**
Austria	1,03				**Slovakia**	1,03			
Capital	890218,7	790245,3	-99973,4	**-11,23%**	**Capital**	314227,1	314227,1	0,0	**0,00%**
Labor	4069,2	3467,5	-601,7	**-14,79%**	**Labor**	2169,8	1528,3	-641,5	**-29,57%**
Emissions	84594,0	74044,5	-10549,5	**-12,47%**	**Emissions**	45982,0	31398,4	-14583,6	**-31,72%**
GDP	258522,9	266813,4	8290,5	**3,21%**	**GDP**	97459,8	103144,2	5684,4	**5,83%**
Inequality	73,9	76,3	2,4	**3,21%**	**Inequality**	74,1	76,3	2,2	**2,93%**
Finland	1,0				**Sweden**	1,0			
Capital	391773,9	391773,9	0,0	**0,00%**	**Capital**	901324,4	901324,4	0,0	**0,00%**
Labor	2482,0	2482,0	0,0	**0,00%**	**Labor**	4509,1	4509,1	0,0	**0,00%**
Emissions	74556,0	74556,0	0,0	**0,00%**	**Emissions**	66232,0	66232,0	0,0	**0,00%**
GDP	149908,6	149908,6	0,0	**0,00%**	**GDP**	284626,6	284626,6	0,0	**0,00%**
Inequality	74,6	74,6	0,0	**0,00%**	**Inequality**	75,9	75,9	0,0	**0,00%**

The projection values give us in information by how much the country has to increase its specific output (and reduce the inputs) to reach the efficiency frontier. The projections from the current data for Germany, Finland and Sweden, countries lying already at the efficiency frontier are equal zero. We can see that for example Greece should increase its GDP and equality indicator by 13,63% to be on the efficiency frontier. Austria is much closer to the frontier and it has to increase its outputs by 3,21% to achieve it. Slovakia is performing worse in terms of GDP and it has to increase it by 5,83% while it has to increase the equality just by 2,93%. This result shows once more the weakness of Slovakia in technical efficiency.

In Table 6 we present the results from **Model 5 (**identical with **Model I)** and **Model 6** and of the **Model 7** that sets inputs to one and takes the scores from these models as outputs. The differences in ecological efficiency (measured by **Model 5)** that considers emissions like an input and GDP as an output are relatively high. An average efficiency score in **Model 5** is 0,61. Switzerland, Malta, France and Germany define the frontier in **Model 5** while in countries like Slovakia, Czech Republic and Cyprus the efficiency score is below 0,4 Socio-economic efficiency is measured by **Model 6** and is presented in the second column of the Table 6. There are more effective countries in this model and the average efficiency score is 0,958. Even though Malta has a score 1 it is just weakly efficient because of excess of capital. The efficiency score in composed model (column 3) cannot be lower than the score in **Model 5** or **Model 6.**

Table 5: Efficiency scores of European countries for the year 2010 (Model 5 - 7)

Model 5 *Ecological efficiency*		Model 6 *Socio economic efficiency*		Model 7 *Composed Model*	
DMU	Score	DMU	Score	DMU	Score
Switzerland	1	Norway	1	Switzerland	1
Malta	1	Belgium	1	Belgium	1
France	1	Iceland	1	Norway	1
Germany	1	United Kingdom	1	Iceland	1
United Kingdom	0.940	Denmark	1	Denmark	1
Spain	0.934	Germany	1	Germany	1
Italy	0.903	Finland	1	United Kingdom	1
Sweden	0.869	Slovenia	1	Finland	1
Norway	0.752	Malta	1	Slovenia	1
Netherlands	0.707	Luxembourg	1	Malta	1
Austria	0.675	France	1	France	1
Portugal	0.611	Sweden	0.996	Luxembourg	1
Belgium	0.600	Netherlands	0.994	Sweden	0.996
Luxembourg	0.568	Hungary	0.994	Netherlands	0.994
Denmark	0.552	Poland	0.993	Hungary	0.994
Iceland	0.510	Czech Republic	0.983	Poland	0.993
Greece	0.499	Slovakia	0.972	Czech Republic	0.983
Romania	0.493	Italy	0.970	Slovakia	0.972
Latvia	0.480	Austria	0.969	Italy	0.970
Hungary	0.473	Cyprus	0.952	Austria	0.969
Ireland	0.447	Spain	0.934	Cyprus	0.952
Poland	0.436	Switzerland	0.925	Spain	0.934
Lithuania	0.433	Estonia	0.914	Estonia	0.914
Slovenia	0.428	Ireland	0.908	Ireland	0.908
Finland	0.425	Portugal	0.895	Portugal	0.895
Slovakia	0.396	Greece	0.880	Greece	0.880
Cyprus	0.378	Bulgaria	0.878	Bulgaria	0.878
Czech Republic	0.375	Romania	0.875	Romania	0.875
Bulgaria	0.258	Lithuania	0.851	Lithuania	0.851
Estonia	0.200	Latvia	0.848	Latvia	0.848

The main aim following by the decomposition of overall efficiency into ecological and socio-economic efficiency is the analysis of the change in efficiency over time which will be discussed shortly in the next Section and will be the subject of further work.

4 Decomposition of the changes in efficiency over time

Besides the efficiency analysis for one particular point of time it is important to investigate the efficiency change over time and to identify the most important sources of these changes. Usually,

total factor productivity, expressed as the ratio of the weighted sum of all the outputs over the weighted sum of all the inputs is used in the general (overall) multiple-input- and multiple-output situations. In multiple-input and multiple-output setting, the application of partial productivity measures often yields misleading results because of the open problem of aggregation of individual indicators and is therefore considered to be not sufficient. Mahlberg – Luptacik – Sahoo (2011) presented the measure of total factor productivity change that provides the possibility to discriminate between observed and "best practice" or frontier performance and distinguish efficiency change from technical change as possible drivers of productivity growth. They proposed a method that allows us to decompose the Malmquist index based on radial input-oriented DEA models of eco-efficiency into the two sub-indexes: economic productivity and ecological productivity. This decomposition enables one to examine the contributions of economic and ecological factors to the total factor productivity change. In this way, we can distinguish between input-saving and environmental-saving changes as drivers of eco-productivity change. They analyzed 14 countries of the European Union for the period 1995 – 2004. Average eco-productivity growth of 22% is observed, while the estimated contribution of improved use of inputs is 20% and of the reduced emissions of greenhouse gases is 23%. Even though for individual countries the results are mixed, on average, the productivity growth is more driven by environmental-saving changes. The analysis of components of total factor productivity change reveals technical progress as its driver. Moreover, the results indicate that for all countries the driving force behind the technical shift is due to environmental-saving technical change.
In this way, using the Models 5-7 in the next step the analysis will be expanded for social aspects in order to examine the factors behind the changes of eco-efficiency and of socio-economic efficiency over time.

5 Conclusions and further research

The urgent need for measuring the economic performance in terms of welfare beyond GDP requires new approaches taking into account simultaneously economic as well as social and environmental indicators. The aim of the paper was to show the advantages of non-parametric approach and extended DEA models in the situations of multiple inputs and multiple outputs when some of the outputs are undesirable.. Various model specifications allow us to capture different characteristics of economic development and to show different possibilities for economic policy to increase the efficiency. For illustration of the possibilities and of the potential of the proposed methodology the data for 30 European countries has been used. The results show the strengths and weaknesses of given countries in different indicators.
Given a set of indicators and defining different scenario and policy outcomes – as planned by WP 205 – DEA methodology can be applied. As shown in the paper by Bosetti – Buchner (2009) devoted to DEA analysis of different climate policy scenarios this methodology allows „to bridge the gap between the simulation phase, in which long-run effects of policies are mimicked, and the valuation phase in which usually a coherent cost benefit analysis framework is adopted." (p. 1342). In difference to the standard application of DEA for the ex-post performance assessment in the proposed approach DEA can be used for ex-ante assessment of different policy scenarios. In order to analyze the impact of different strategies and goals of economic policy, DEA models with restricted weights can be used. Therefore as a next step for further research the restrictions for the weights (dual variables in our DEA models) will be introduced and the impact of their change for the measuring of social welfare investigated. The weights for specific goals can be estimated by using interactive methods of MCA, in particularly methodology known as Analytic Hierarchy Process. In this way, human judgments and priorities that appear to be crucial for the increasing well-being of the population may be taken into account. As a consequence, the results obtained by adopting the methodology proposed in this paper can provide an important decision support for policy makers.

6 References

[1] Acemoglu, D. 2009. *Introduction to Modern Economic Growth.* Princeton University Press: Princeton and Oxford, 2009.

[2] Banker, R. – Charnes, A. – Cooper, W. 1984. *Some models for estimating technical and scale inefficiences in data envelopment analysis*. Management Science, 30 (1984), 1078–1092.

[3] Bosetti, V. – Buchner, B. 2009. *Data Envelopment Analysis of different climate policy scenarios*. Ecological Economics 68 [2009], 1340-1354.

[4] Charnes, A. – Cooper, W. – Rhodes, E. 1978. *Measuring the efficiency of decision making units*. European Journal of Operational Research, 2 (1978), 429–444.

[5] Charnes, A. – Cooper, W. – Rhodes, E. 1979. *Short communication: Measuring the efficiency of decision making units.* European Journal of Operational Research, 3 (1979), 339.

[6] Charnes, A. – Cooper, W. 1962. *Programming with Linear Fractional Functionals*. Naval Research Logistics Quarterly 9, 181–196.

[7] Dujava, D. 2012. *Causes of Lagging Behind of New Member States of EU: Empirical Analysis by Montgomery Decomposition*. Politická ekonomie, University of Economics, Prague, vol. 2012(2), p. 222-244.

[8] Dyckhoff, H. – Allen, K. 2001. *Measuring ecological efficiency with data envelopment analysis (DEA).* European Journal of Operational Research, 132 (2001), 312–325.

[9] European Commission. 2005. *Communication from the commission to the Council and the European Parliament.* 2004 Environmental Policy Review, Brussels, 27-1-2005.

[10] Färe, R. – Grosskopf, S. – Lovell, C. – Pasurka, C. 1989. *Multilateral productivity comparison when some outputs are undesirable: A nonparametric approach*. Review of Economics and Statistics, 71 (1989), 90–98.

[11] Färe, R. – Grosskopf, S. – Tyteca, D. 1996. *An activity analysis model of the environmental performance of firms: Application to fossil-fuel-fired electric utilities.* Ecological Economics, 18 (1996), 161–175.

[12] Fried, H. O.et al. 2008. The *Measurement of Productive Efficiency and Productivity Growth*. Oxford University Press, USA, 2008.

[13] Griliches, Z. 1995. *The Discovery of the Residual: An Historical Note*. National Bureau of Economic Research: Cambridge, NBER Working Paper Series, WP No. 5348, November 1995.

[14] Korhonen, P. – Luptáčik, M. 2004. *Eco-efficiency analysis of power plants: An extension of data envelopment analysis*. European Journal of Operational Research, 154 (2004), 437–446

[15] Mahlberg, B. – Luptáčik, M. – Sahoo, B.K. 2011. *Examining the drivers of total factor productivity change with an illustrative example of 14 EU countries*. Ecological Economics 72 [2011] 60-69.

[16] OECD. 1998. *Eco-efficiency.* Paris: Organization for Economic Co-operation and Development, 1998.

[17] Rao, P. – Coelli, T.J. 1999. *Economic Growth, Productivity Change and Inequality: Methodology for the Assesment of Economic Performance of Nations*. Working paper for the Workshop at the Kanda Campus of Senshu University, August 24, 1999.

[18] Schaltegger, St. – Sturm, A. 1989. *Ökologieinduzierte Entscheidungsprobleme des Managements, Ansatzpunkte zur Ausgestaltung von Instrumenten.* WWZ-Discussion Paper No. 8914, WWZ, Basel.

[19] Solow, R. 1957. *Technical change and the aggregate production function*. Review of Economics and Statistics 39(3): 312-320.

[20] ten Raa, Th. , Mohnen P. 2002. Neoclassical *growth accounting and frontier analysis: a synthesis*. Journal of Productivity Analysis, Vol. 18(2): 111-128

[21] Tyteca, D. 1996. *On the measurement of the environmental performance of firms: A literature review and a productivity efficiency perspective.* Journal of Environmental Management, 46 (1996), 281–308.

[22] Tyteca, D. 1997. *Linear programming models for the measurement of environmental performance of firms: Concepts and empirical results*. Journal of Productivity Analysis, 8 (1997), 183–197.

[23] WBCSD .1996. E*co-efficient Leadership for Improved Economic and Environmental Performance*